2

Present Value

The Mother of All Finance

We begin with the concept of a rate of return — the cornerstone of finance. You can always earn an interest rate (and interest rates are rates of return) by depositing your money today into the bank. This means that money today is more valuable than the same amount of money next year. This concept is called the *time value of money* (TVM) — \$1 in present value is better than \$1 in future value.

Investors make up just one side of the financial markets. They give money today in order to receive money in the future. Firms often make up the other side. They decide what to do with the money — which projects to take and which projects to pass up — a process called *capital budgeting*. You will learn that there is one best method for making this critical decision. The firm should translate all *future* cash flows — both inflows and outflows — into their equivalent *present values* today. Adding in the cash flow today gives the *net present value*, or NPV. The firm should take all projects that have positive net present values and reject all projects that have negative net present values.

This all sounds more complex than it is, so we'd better get started.

2.1 The Basic Project Scenario

As promised, we begin with the simplest possible scenario. In finance, this means that we assume that we are living in a so-called perfect market:

- There are no taxes.
- There are no transaction costs (costs incurred when buying and selling).
- There are no differences in information or opinions among investors (although there can be risk).
- There are so many buyers and sellers (investors and firms) in the market that the presence or absence of just one (or a few) individuals does not have an influence on the price.

The perfect market allows us to focus on the basic concepts in their purest forms, without messy real-world factors complicating the exposition. We will use these assumptions as our sketch of how financial markets operate, though not necessarily how firms' product markets work. You will learn in Chapter how to operate in a world that is not perfect. (This will be a lot messier.)

In this chapter, we will make three additional assumptions (that are not required for a market to be considered "perfect") to further simplify the world:

- The interest rate per period is the same.
- There is no inflation.
- There is no risk or uncertainty. You have perfect foresight.

Of course, this financial utopia is unrealistic. However, the tools that you will learn in this chapter will also work in later chapters, where the world becomes not only progressively more realistic but also more difficult. Conversely and more importantly, if any tool does not give the right answer in our simple world, it would surely make no sense in a more realistic one. And, as you will see, the tools have validity even in the messy real world.

Begin Important

- Tools that do not work in simple scenarios will almost surely *not* work in more realistic scenarios. This is why simple scenarios are so useful.
- However, tools that work in simple scenarios may not work in more realistic scenarios.

End Important

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2.2 Loans and Bonds

The material in this chapter is easiest to explain in the context of bonds and loans. A loan is the commitment of a borrower to pay a predetermined amount of cash at one or more predetermined times in the future (the final one called maturity), usually in exchange for cash upfront today. Loosely speaking, the difference between the money lent and the money paid back is the interest that the lender earns. A bond is a particular kind of loan, so named because it “binds” the borrower to pay money. Thus, for an investor, “buying a bond” is the same as “extending a loan.” Bond buying is the process of giving cash today and receiving a binding promise for money in the future. Similarly, from the firm’s point of view, it is “giving a bond,” “issuing a bond,” or “selling a bond.” Loans and bonds are also sometimes called fixed income securities, because they promise a fixed amount of payments to the holder of the bond.

You should view a bond as just another type of investment project — money goes in, and money comes out. You could slap the name “corporate project” instead of “bond” on the cash flows in the examples in this chapter, and nothing would change. In Chapter , you will learn more about Treasuries, which are bonds issued by the U.S. Treasury. The beauty of such bonds is that you know exactly what your cash flows will be. (Despite political dysfunction in Washington DC, we will assume that our Treasury cannot default.) Besides, much more capital in the economy is tied up in bonds and loans than is tied up in stocks, so understanding bonds well is very useful in itself.

You already know that the net return on a loan is called interest, and that the rate of return on a loan is called the interest rate — though we will soon firm up your knowledge about interest rates. One difference between an interest payment and a noninterest payment is that the former usually has a maximum payment, whereas the latter can have unlimited upside potential. However, not every rate of return is an interest rate. For example, an investment in a lottery ticket is not a loan, so it does not offer an interest rate, just a rate of return. In real life, its payoff is uncertain — it could be anything from zero to an unlimited amount. The same applies to stocks and many corporate projects. Many of our examples use the phrase "interest rate," even though the examples almost always work for any other rates of return, too.

Is there any difference between buying a bond for $1,000 and putting $1,000 into a bank savings account? Yes, a small one. The bond is defined by its future promised payoffs — say, $1,100 next year — and the bond's value and price today are based on these future payoffs. But as the bond owner, you know exactly how much you will receive next year. An investment in a bank savings account is defined by its investment today. The interest rate can and will change every day, so you do not know what you will end up with next year. The exact amount depends on future interest rates. For example, it could be $1,080 (if interest rates decrease) or $1,120 (if interest rates increase).

If you want, you can think of a savings account as a sequence of consecutive 1-day bonds: When you deposit money, you buy a 1-day bond, for which you know the interest rate this one day in advance, and the money automatically gets reinvested tomorrow into another bond with whatever the interest rate will be tomorrow.

(Omitted solvenow)

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2.3 Returns, Net Returns, and Rates of Return

The most fundamental financial concept is that of a return. The payoff or (dollar) return of an investment is simply the amount of cash (C) it returns. For example, an investment project that returns $12 at time 1 has

(Omitted eq)

This subscript is an instant in time, usually abbreviated by the letter t. When exactly time 1 occurs is not important: It could be tomorrow, next month, or next year. But if we mean "right now," we use the subscript 0.

The net payoff, or net return, is the difference between the return and the initial investment. It is positive if the project is profitable and negative if it is unprofitable. For example, if the investment costs $10 today and returns $12 at time 1 with nothing in between, then it earns a net return of $2. Notation-wise, we need to use two subscripts on returns — the time when the investment starts (0) and when it ends (1).

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The double subscripts are painful. Let's agree that if we omit the first subscript on flows, it means zero. The rate of return, usually abbreviated r, is the net return expressed as a percentage of the initial investment.

(Omitted eq)

Here, I used our new convention and abbreviated $r_{0,1}$ as r_1. Often, it is convenient to calculate the rate of return as

(Omitted eq)

Percent (the symbol %) is a unit of 1/100. So 20% is the same as 0.20.

(Omitted anecdote)

Many investments have interim payments. For example, many stocks pay interim cash dividend[dividends], many bonds pay interim cash coupon[coupons], and many real estate investments pay interim rent. How would you calculate the rate of return then? One simple method is to just add interim payments to the numerator. Say an investment costs $92, pays a dividend of $5 (at the end of the period), and then is worth $110. Its rate of return is

(Omitted eq)

When there are intermittent and final payments, then returns are often broken down into two additive parts. The first part, the price change or capital gain, is the difference between the purchase price and the final price, *not* counting interim payments. Here, the capital gain is the difference between $110 and $92, that is, the $18 change in the price of the investment. It is often quoted in percent of the price, which would be $18/$92 or 19.6% here. The second part is the amount received in interim payments. It is the dividend or coupon or rent, here $5. When it is divided by the price, it has names like dividend yield, current yield, rental yield, or coupon yield, and these are also usually stated in percentage terms. In our example, the dividend yield is $\$5/\$92 \approx 5.4\%$. Of course, if the interim yield is high, you might be experiencing a negative capital gain and still have a positive rate of return. For example, a bond that costs $500, pays a coupon of $50, and then sells for $490, has a capital loss of $10 (which comes to a –2% capital yield) but a rate of return of $(\$490 + \$50 - \$500)/\$500 = +8\%$. You will almost always work with rates of return, not with capital gains. The only exception is when you have to work with taxes, because the IRS treats capital gains differently from interim payments. (We will cover taxes in Section .) 2]sect:introtaxesTaxes on capital gains

Most of the time, people (incorrectly but harmlessly) abbreviate a rate of return or net return by calling it just a return. For example, if you say that the return on your $10,000 stock purchase was 10%, you obviously do not mean you received a unitless 0.1. You really mean that your rate of return was 10% and you received $1,000. This is usually benign, because your listener will know what you mean. Potentially more harmful is the use of the phrase *yield*, which, strictly speaking, means *rate of return*. However, it is often misused as a shortcut for dividend yield or coupon yield (the percent payout that a stock or a bond provides). If you say that the yield on your stock was 5%, then some listeners may interpret it to mean that you earned a total rate of return of 5%, whereas others may interpret it to mean that your stock paid a dividend yield of 5%.

2]sect:apples-nominal-inflationNominal

Interest rates should logically always be positive. After all, you can always earn 0% if you keep your money under your mattress — you thereby end up with as much money next period as you have this period. Why give your money to someone today who will give you less than 0% (less money in the future)? Consequently, interest rates are indeed almost always positive — the rare exceptions being both bizarre and usually small (accounting for the cost of safekeeping cash).

(Omitted anecdote)

Here is another language problem: What does the statement "the interest rate has just increased by 5%" mean? It could mean either that the previous interest rate, say, 10%, has just increased from 10% to $10\% \cdot (1 + 5\%) = 10.5\%$, or that it has increased from 10% to 15%. Because this is unclear, the basis point unit was invented. A basis point is simply 1/100 of a percent and often abbreviated as bps, pronounced "bips." If you state that your interest rate has increased by 50 basis points, you definitely mean that the interest rate has increased from 10% to 10.5%. If you state that your interest rate has increased by 500 basis points, you definitely mean that the interest rate has increased from 10% to 15%.

Begin Important

100 basis points constitute 1%. Somewhat less common, 1 point is 1%.

Points and basis points help with "percentage ambiguities." **End Important**

(Omitted solvenow)

2.4 Time Value, Future Value, and Compounding

Because you can earn interest, a given amount of money today is worth more than the same amount of money in the future. After all, you could always deposit your money today into the bank and thereby receive more money in the future. This is an example of the time value of money, which says that a dollar today is more useful today than a dollar tomorrow. This ranks as one of the most basic and important concepts in finance.

The Future Value of Money

How much money will you receive in the future if the rate of return is 20% and you invest $100 today? Turn around the rate of return formula (Formula **??**)]eq:retRate of Return to determine how money will grow over time given a rate of return:

(Omitted eq)

The $120 next year is called the future value (FV) of $100 today. Thus, future value is the value of a present cash amount at some point in the future. It is the time value of money that causes the future value, $120, to be higher than its present value (PV), $100. Using the abbreviations FV and PV, you could also have written the above formula as

(Omitted eq)

(If we omit the subscript on the r, it means a 1-period interest rate from now to time 1, i.e., r_1.) Please note that the time value of money is not the fact that the prices of goods may change between today and tomorrow (that would be inflation). Instead, the time value of money is based exclusively on the fact that your money can earn interest. Any amount of cash today is worth more than the same amount of cash tomorrow. Tomorrow, it will be the same amount plus interest.

(Omitted solvenow)

Compounding and Future Value

Now, what if you can earn the same 20% year after year and reinvest all your money? What would your two-year rate of return be? Definitely *not* $20\% + 20\% = 40\%$! You know that you will have \$120 in year 1, which you can reinvest at a 20% rate of return from year 1 to year 2. Thus, you will end up with

(Omitted eq)

This \$144 — which is, of course, again a future value of \$100 today — represents a total two-year rate of return of

(Omitted eq)

This is more than 40% because the original net return of \$20 in the first year earned an additional \$4 in interest in the second year. You earn interest on interest! This is also called compound interest. Similarly, what would be your 3-year rate of return? You would invest \$144 at 20%, which would provide you with

(Omitted eq)

Your 3-year rate of return from time 0 to time 3 (call it r_3) would thus be

(Omitted eq)

This formula translates the three sequential 1-year rates of return into one 3-year holding rate of return — that is, what you earn if you hold the investment for the entire period. This process is called compounding, and the formula that does it is the "one-plus formula":

(Omitted eq)

or, if you prefer it shorter, $1.728 = 1.2^3$. (Compound return and compounded return mean the same thing.)

Begin Important

The formula to translate sequential future rates of return into one overall holding rate of return is

(Omitted eq)

each omitting the "–1." The first rate is called the spot rate because it starts now (on the spot).

This compounding formula is so common that you must memorize it. End Important

Figure **??** shows how your \$100 would grow if you continued investing it at a rate of return of 20% per annum. The function is exponential — that is, it grows faster and faster as interest earns more and more interest.

(Omitted fig)

You can use the compounding formula to compute all sorts of future payoffs. For example, an investment project that costs \$212 today and earns 10% each year for 12 years will yield an overall holding rate of return of

(Omitted eq)

Your \$212 investment today would therefore turn into a future value of

(Omitted eq)

Now suppose you wanted to know what constant two 1-year interest rates (r) would give you a two-year rate of return of 50%. The answer is not 25%, because $(1 + 25\%) \cdot (1 + 25\%) - 1 = 1.25^2 - 1 = 56.25\%$. Instead, you need to solve

(Omitted eq)

The correct answer is

(Omitted eq)

2]ch:appendixExponentiation, Book AppendixCheck your answer: $(1 + 22.47\%) \cdot (1 + 22.47\%) = 1.2247^2 \approx (1 + 50\%)$. If the 12-month interest rate is 213.8%, what is the 1-month interest rate?

(Omitted eq)

Interestingly, compounding works even over fractional time periods. Say the overall interest rate is 5% per year, and you want to find out what the rate of return over half a year would be. Because $(1 + r_{0.5})^2 = (1 + r_1)$, you would compute

(Omitted eq)

Check — compounding 2.4695% over two (6-month) periods indeed yields 5%:

(Omitted eq)

(Omitted anecdote)

If you know how to use logarithms, you can also use the same formula to determine how long it will take at the current interest rate to double or triple your money. For example, at an interest rate of 3% per year, how many years would it take you to double your money?

(Omitted eq)

Compound rates of return can be negative even when the average rate of return is positive: think +200% followed by –100%. The average arithmetic rate of return in this example is $(200\% + (-100\%))/2 = +50\%$, while the compound rate of return is –100%. Not a good investment! Thinking in arithmetic terms for wealth accumulation is a common mistake, if only because funds usually advertise their average rate of return. High-volatility funds (i.e., funds that increase and decrease a lot in value) look particularly good on this incorrect performance measure.

Errors: Adding or Compounding Interest Rates?

Unfortunately, when it comes to interest rates in the real world, many users are casual, sometimes to the point where they are outright wrong. Some people mistakenly add interest rates instead of compounding them. When the investments, interest rates, and time length are small, the difference between the correct and incorrect computation is often minor, so this practice can be acceptable, even if it is wrong. For example, when interest rates are 10%, compounding yields

(Omitted eq)

the same as $2 \cdot r + r \cdot r$. This is not exactly the same as the simple sum of two r's, which comes to 20%. The difference between 21% and 20% is the "cross-term" $r \cdot r$. This cross-product is especially unimportant if both rates of return are small. If the two interest rates were both 1%, the cross-term would be 0.0001. This is indeed small enough to be ignored in most situations and is therefore a forgivable approximation. However, when you compound over many periods, you accumulate more and more cross-terms, and eventually the quality of your approximation deteriorates. For example, over 100 years, $1 million invested at 1% per annum compounds to $\$1 \cdot 1.01^{100} \approx \$2.70$ million, not to $2 million.

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How Banks Quote Interest Rates

Banks and many other financial institutions use a number of conventions for quoting interest rates that may surprise you. Consider the example of a loan or a deposit that has one flow of $1,000,000 and a return flow of $1,100,000 in six months. Obviously, the simple holding rate of return is 10%. Here is what you might see:

The effective annual rate (EAR) is what our book has called the "real-deal" interest rate or holding rate of return. In this case, our only problem is to re-quote the six-month 10% rate into a twelve-month rate. This is easy,

(Omitted eq)

This 21% is usually a supplementary rate that any bank would quote you on both deposits and loans. The EAR is also sometimes called the annual percentage yield (APY). And it is also sometimes (and ambiguously) called the annual equivalent rate (AER).

The annual interest rate (stated without further explanations) is not really a rate of return, but just a method of quoting an interest rate. The true daily interest rate is this annual interest quote divided by 365 (or 360 by another convention). In the example, the 10% half-year interest rate translates into

(Omitted eq)

The annual interest rate is usually (but not always) how banks quote interest rates on savings or checking accounts. Conversely, if the bank advertises a savings interest rate of 20%, any deposit would really earn an effective annual rate of $(1 + 20\%/365)^{365} - 1 \approx 22.13\%$ per year.

The annual percentage rate (APR) is a complete mess. Different books define it differently. Most everyone agrees that APR is based on monthly compounding:

(Omitted eq)

However, the APR is also supposed to include fees and other expenses. Say the bank charged $10,000 in application and other fees. This is paid upfront, so we should recognize that the interest rate is not 10%, but $1,100,000/$990,000 – 1 ≈ 11.1%. We could then "monthly-ize" this holding rate of return into an APR of $(1.111^{2/12} - 1) \cdot 12 \approx 21.26\%$.

So far, so good — except different countries require different fees to be included. In the United States, there are laws that state how APR should be calculated — and not just one, but a few (the Truth in Lending Act of 1968 [Reg Z], the Truth in Savings Act of 1991, the Consumer Credit Act of 1980, and who knows what other Acts). Even with all these laws, the APR is still not fully precise and comparable. To add insult to injury, the APR is also sometimes abbreviated as AER, just like the EAR.

Interest rates are not intrinsically difficult, but they can be tedious, and definitional confusions abound. So if real money is on the line, you should ask for the full and exact calculations of all payments in and all payments out, and not just rely on what *you* think it is that the bank is really quoting you. Besides, the above rates are not too interesting (yet), because they don't work for loans that have multiple payments. You have to wait for that until we cover the yield-to-maturity.2]sect:IRRandYTMYield-To-Maturity

Let's look at a certificate of deposit (CD), which is a longer-term investment vehicle than a savings account deposit. If your bank wants you to deposit your money in a CD, do you think it will put the more traditional interest rate quote or the APY on its sign in the window? Because the APY looks larger and thus more appealing to depositors than the traditional interest rate quote (APR), most banks advertise the APY for deposits. If you want to borrow money from your bank, do you think your loan agreement will similarly emphasize the APY? No. Most of the time, banks leave this number to the fine print and focus on the APR (or the traditional interest rate quote) instead.

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2.5 Present Value, Discounting, and Capital Budgeting

Now turn to the flip side of the future value problem: If you know how much money you will have next year, what does this correspond to in value *today*? This is especially important in a corporate context, where the question is, "Given that Project X will return $1 million in 5 years, how much should you be willing to pay to undertake this project today?" The process entailed in answering this question is called capital budgeting and is at the heart of corporate decision making. (The origin of the term was the idea that firms have a "capital budget," and that they must allocate capital to their projects within that budget.)

Start again with the rate of return formula

$$r_1 = \frac{C_1 - C_0}{C_0} = \frac{C_1}{C_0} - 1$$

You only need to turn this formula around to answer the following question: If you know the prevailing interest rate in the economy (r_1) and the project's future cash flows (C_1), what is the project's value to you *today*? In other words, you are looking for the present value (PV) — the amount a future sum of money is worth today, given a specific rate of return. For example, if the interest rate is 10%, how much would you have to save (invest) to receive

The answer lies in the present value formula, which translates future money into today's money. You merely need to rearrange the rate of return formula to solve for the present value:

(Omitted eq)

Check this — investing \$90.91 at an interest rate of 10% will indeed return \$100 next period:

(Omitted eq)

This is the present value formula, which uses a division operation known as discounting. (The term "discounting" indicates that we are reducing a value, which is exactly what we are doing when we translate future cash into current cash.) If you wish, you can think of discounting — the conversion of a future cash flow amount into its equivalent present value amount — as the *reverse* of compounding.

Thus, the present value (PV) of next year's \$100 is \$90.91 — the value today of future cash flows. Let's say that this \$90.91 is what the project costs. If you can borrow or lend at the interest rate of 10% elsewhere, then you will be indifferent between receiving \$100 next year and receiving \$90.91 for your project today. In contrast, if the standard rate of return in the economy were 12%, your specific project would not be a good deal. The project's present value would be

(Omitted eq)

which would be less than its cost of \$90.91. But if the standard economy-wide rate of return were 8%, the project would be a great deal. Today's present value of the project's future payoff would be

(Omitted eq)

which would exceed the project's cost of \$90.91. It is the present value of the project, weighed against its cost, that should determine whether you should undertake a project today or avoid it. The present value is also the answer to the question, "How much would you have to save at current interest rates today if you wanted to have a specific amount of money next year?"

Let's extend the time frame in our example. If the interest rate were 10% per period, what would \$100 in two periods be worth today? The value of the \$100 is then

(Omitted eq)

Note the 21%. In two periods, you could earn a rate of return of $(1 + 10\%) \cdot (1 + 10\%) - 1 = 1.1^2 - 1 = 21\%$ elsewhere, so this is your appropriate comparable rate of return.

This discount rate — the rate of return, r, with which the project can be financed — is often called the cost of capital. It is the rate of return at which you can raise money elsewhere. In a perfect market, this cost of capital is also the opportunity cost that you bear if you fund your specific investment project instead of the alternative next-best investment elsewhere. Remember — you can invest your money at this opportunity rate in another project instead of this one. When these alternative projects in the economy elsewhere are better, your cost of capital is higher, and the value of your specific investment project with its specific cash flows is relatively lower. An investment that promises $1,000 next year is worth less today if you can earn 50% rather than 5% elsewhere. A good rule is to always mentally add the word "opportunity" before "cost of capital" — it is always your opportunity cost of capital. (In this part of our book, I will just tell you what the economy-wide rate of return is — here 10% — for borrowing or investing. In later chapters, you will learn how this opportunity cost of capital [ahem, "rate of return"] is determined.)2]sect:hurdlerateImperfect-Market Hurdle Rates

Begin Important

Always think of the r in the present value denominator as your "opportunity" cost of capital. If you have great opportunities elsewhere, your projects have to be discounted at high discount rates. The discount rate, the cost of capital, and the required rate of return are really all just different names for the same thing in a perfect market. **End Important**

When you multiply a future cash flow by its appropriate discount factor, you end up with its present value. Looking at Formula **??**, you can see that this discount factor is the quantity

(Omitted eq)

In other words, the discount factor translates 1 dollar in the future into the equivalent amount of dollars today. In the example, at a two-year 21% rate of return, a dollar in two years is worth about 83 cents today. Because interest rates are usually positive, discount factors are usually less than 1 — a dollar in the future is worth less than a dollar today. (Sometimes, people call this the discount rate, but the discount rate is really $r_{0,t}$ if you are a stickler for accuracy.)

(Omitted fig)

Figure **??** shows how the discount factor declines when the cost of capital is 20% per annum. After about a decade, any dollar the project earns is worth less than 20 cents to you today. If you compare Figure **??** to Figure **??**, you should notice how each is the "flip side" of the other.

Begin Important

The cornerstones of finance are the following formulas:

(Omitted eq)

Rearrange the formula to obtain the future value:

(Omitted eq)

The process of obtaining $r_{0,t}$ is called compounding, and it works through the "one-plus" formula:

(Omitted eq)

Rearrange the formula again to obtain the present value:

(Omitted eq)

The process of translating C_t into C_0 — that is, the multiplication of a future cash flow by $1/(1 + r_{0,t})$ — is called discounting. The discount factor is:

(Omitted eq)

It translates one dollar at time t into its equivalent value today. **End Important**

Remember how bonds are different from savings accounts? The former is pinned down by its promised fixed future payments, while the latter pays whatever the daily interest rate is. This induces an important relationship between the value of bonds and the prevailing interest rates — *they move in opposite directions.* For example, if you have a bond that promises to pay $1,000 in one year, and the prevailing interest rate is 5%, the bond has a present value of $\$1{,}000/1.05 \approx \$952.38$. If the prevailing interest rate suddenly increases to 6% (and thereby becomes your new opportunity cost of capital), the bond's present value becomes $\$1{,}000/1.06 \approx \$943.40$. You lose $8.98, which is about 0.9% of your original $952.38 investment. The value of your fixed-bond payment in the future has gone down, because investors can now do better than your 5% by buying new bonds. They have better opportunities elsewhere in the economy. They can earn a rate of return of 6%, not just 5%, so if you wanted to sell your bond now, you would have to sell it at a discount to leave the next buyer a rate of return of 6%. If you had delayed your investment, the sudden change to 6% would have done nothing to your investment. On the other hand, if the prevailing interest rate suddenly drops to 4%, then your bond will be more valuable. Investors would be willing to pay $\$1{,}000/1.04 \approx \$961.54$, which is an immediate $9.16 gain. The inverse relationship between prevailing interest rates and bond prices is general and worth noting.

Begin Important

The price and the implied rate of return on a bond with fixed payments move in opposite directions. When the price of the bond goes up, its implied rate of return goes down. When the price of the bond goes down, its implied rate of return goes up. **End Important**

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2.6 Net Present Value

An important advantage of present value is that all cash flows are translated into the same unit: cash today. To see this, say that a project generates \$10 in one year and \$8 in five years. You cannot add up these different future values to come up with \$18 — it would be like adding apples and oranges. However, if you translate both future cash flows into their present values, you *can* add them. For example, if the interest rate was 5% per annum (so $(1 + 5\%)^5 = (1 + 27.6\%)$ over 5 years), the present value of these two cash flows together would be

(Omitted eq)

Therefore, the total value of the project's future cash flows *today* (at time 0) is \$15.79.

The net present value (NPV) of an investment is the present value of all its future cash flows minus the present value of its cost. It is really the same as present value, except that the word "net" upfront reminds you to add and subtract *all* cash flows, including the *upfront* investment outlay today. The NPV calculation method is always the same:

1. Translate all future cash flows into today's dollars.
2. Add them all up. This is the present value of all future cash flows.
3. Subtract the initial investment.

NPV is the most important method for determining the value of projects. It is a cornerstone of finance. Let's assume that you have to pay \$12 to buy this particular project with its \$10 and \$8 cash flows. In this case, it is a positive NPV project, because

(Omitted eq)

(For convenience, we omit the 0 subscript for NPV, just as we did for PV.)

There are a number of ways to understand net present value.

- One way is to think of the NPV of \$3.79 as the difference between the market value of the future cash flows (\$15.79) and the project's cost (\$12) — this difference is the "value added."
- Another way to think of your project is to compare its cash flows to an equivalent set of bonds that exactly *replicates* them. In this instance, you would want to purchase a 1-year bond that promises \$10 next year. If you save \$9.52 — at a 5% interest rate — you will receive \$10. Similarly, you could buy a 5-year bond that promises \$8 in year 5 for \$6.27. Together, these two bonds exactly replicate the project cash flows. The law of one price tells you that your project should be worth as much as this bond project — the cash flows are identical. You would have had to put away \$15.79 today to buy these bonds, but your project can deliver these cash flows at a cost of only \$12 — much cheaper and thus better than your bond alternative.
- There is yet another way to think of NPV. It tells you how your project compares to the alternative opportunity of investing in the capital markets. These opportunities are expressed in the denominator through the discount factor. What would you get if you took your \$12 and invested it in the capital markets instead of in your project? Using the future value formula, you know that you could earn a 5% rate of return from now

to next year, and 27.6% from now to 5 years. Your $12 would grow into $12.60 by next year. You could take out the same $10 cash flow that your project gives you and be left with $2.60 for reinvestment. Over the next 4 years, at the 5% interest rate, this $2.60 would grow into $3.16. But your project would do better for you, giving you $8. Thus, your project achieves a higher rate of return than capital markets alternatives.

The conclusion of this argument is not only the simplest but also the best capital budgeting rule: If the NPV is positive, as it is for our $3.79 project, take the project. If it is negative, reject the project. If it is zero, it does not matter.

Begin Important

- The NPV formula is

  (Omitted eq)

  The subscripts are time indexes, C_t is the net cash flow at time t (positive for inflows, negative for outflows), and r_t is the relevant interest rate for investments from now to time t. With constant interest rates, $r_t = (1+r)^t - 1$.

- The NPV capital budgeting rule states that you should accept projects with a positive NPV and reject those with a negative NPV.
- Taking positive NPV projects increases the value of the firm. Taking negative NPV projects decreases the value of the firm.
- NPV is definitively the best method for capital budgeting — the process by which you should accept or reject projects.

The NPV formula is so important that you must memorize it.

End Important

Let's work another NPV example. A project costs $900 today, yields $200/year for two years, then $400/year for two years, and finally requires a cleanup expense of $100. The prevailing interest rate is 5% per annum. These cash flows are summarized in Table **??**. Should you take this project?

1. You need to determine the cost of capital for tying up money for one year, two years, three years, and so on. The compounding formula is

   (Omitted eq)

   So for money right now, the cost of capital r_0 is $1.05^0 - 1 = 0$; for money in one year, r_1 is $1.05^1 - 1 = 5\%$; for money in two years, r_2 is $1.05^2 - 1 = 10.25\%$. And so on.

2. You need to translate the cost of capital into discount factors. Recall that these are 1 divided by 1 plus your cost of capital. A dollar in one year is worth $1/(1 + 5\%) = 1/1.05 \approx 0.9524$ dollars today. A dollar in two years is worth $1/(1 + 5\%)^2 = 1/1.05^2 \approx 0.9070$. And so on.

3. You can now translate the future cash flows into their present value equivalents by multiplying the payoffs by their appropriate discount factors. For example, the \$200 cash flow at time 1 is worth about $0.9524 \cdot \$200 \approx \$190.48$.
4. Because present values are additive, you then sum up all the terms to compute the overall net present value. Make sure you include the original upfront cost as a negative.

Consequently, the project NPV is about \$68.15. 1]pg:rounding-error-box\$68.14 or \$68.15?: Rounding Error Because \$68.15 is a positive value, you should take this project.

(Omitted tbl)

However, if the upfront expense were \$1,000 instead of \$900, the NPV would be negative (–\$31.84), and you would be better off investing the money into the appropriate sequence of bonds from which the discount factors were computed. In this case, you should have rejected the project.

(Omitted solvenow)

Application: Are Faster-Growing Firms Better Bargains?

Let's work another NPV problem, applying to companies overall. Does it make more sense to invest in companies that are growing quickly rather than slowly? If you wish, you can think of this question loosely as asking whether you should buy stock of a fast-growing company like Google or stock of a slow-growing company like Procter & Gamble. Actually, you do not even have to calculate anything. In a perfect market, the answer is always that every publicly traded investment comes for a fair price. Thus, the choice does not matter. Whether a company is growing quickly or slowly is already incorporated in the firm's price today, which is just the present value of the firm's cash flows that will accrue to the owners. Therefore, neither is the better deal. Yet, because finance is so much fun, we will ignore this little nuisance and work out the details anyway.

For example, say company "Grow" (G) will produce over the next 3 years

(Omitted eq)

and company "Shrink" (S) will produce

(Omitted eq)

Is G not a better company to buy than S?

There is no uncertainty involved, and both firms face the same cost of capital of 10% per annum. The price of G today is its present value (PV)

(Omitted eq)

and the price of S today is

(Omitted eq)

What is your rate of return from this year to next year? If you invest in G, then next year you will have \$100 cash and own a company with \$150 and \$250 cash flows coming up. G's value at time 1 (so PV now has subscript 1 instead of the usually omitted 0) will thus be

(Omitted eq)

Your investment will have earned a rate of return of \$442.98/\$402.70 – 1 ≈ 10%. If you invest instead in S, then next year you will receive \$100 cash and own a company with "only" \$90 and \$80 cash flows coming up. S's value will thus be

(Omitted eq)

Your investment will have earned a rate of return of \$247.93/\$225.39 – 1 ≈ 10%. In either case, you will earn the fair rate of return of 10% from this year to next year. Whether cash flows are growing at a rate of +50%, -10%, +237.5%, or -92% is irrelevant: *The firms' market prices today already reflect their future growth rates.* There is no necessary connection between the growth rate of the underlying project cash flows or earnings and the growth rate of your investment money (i.e., your expected rate of return).

Make sure you understand the thought experiment here: This statement that higher-growth firms do not necessarily earn a higher rate of return does not mean that a firm in which managers succeed in increasing the future cash flows at no extra investment cost will not be worth more. Such firms will indeed be worth more, and the current owners will benefit from the rise in future cash flows, but this will also be reflected immediately in the price at which you, an outsider, can buy this firm. This is an important corollary worth repeating. If General Electric has just won a large defense contract (like the equivalent of a lottery), shouldn't you purchase GE stock to participate in the windfall? Or if Wal-Mart managers do a great job and have put together a great firm, shouldn't you purchase Wal-Mart stock to participate in this windfall? The answer is that you cannot. The old shareholders of Wal-Mart are no dummies. They know the capabilities of Wal-Mart and how it will translate into cash flows. Why should they give you, a potential new shareholder, a special bargain for something to which you contributed nothing? Just providing more investment funds is not a big contribution — after all, there are millions of other investors equally willing to provide funds at the appropriate right price. It is competition — among investors for providing funds and among firms for obtaining funds — that determines the expected rate of return that investors receive and the cost of capital that firms pay. There is actually a more general lesson here. Economics tells you that you must have a scarce resource if you want to earn above-normal profits. Whatever is abundant and/or provided by many competitors will not be a tremendously profitable business.

An even more general version of the question in this section (whether fast-growing or slow-growing firms are better investments) is whether good companies are better investments than bad companies. Many novices will answer that it is better to buy a good company. But you should immediately realize that the answer must depend on the price. Would you really want to buy a great company if its cost was twice its value? And would you really not want to buy a lousy company if you could buy it for half its value? For an investment, whether a company is a growing purveyor of perfume or a shrinking purveyor of manure does not matter by itself. What matters is only the company price relative to the future discounted company cash flows that you will receive.

(Omitted solvenow)

Summary

This chapter covered the following major points:

- A perfect market assumes no taxes, no transaction costs, no opinion differences, and the presence of many buyers and sellers.
- A bond is a claim that promises to pay an amount of money in the future. Buying a bond is extending a loan. Issuing a bond is borrowing. Bond values are determined by their future payoffs.
- One hundred basis points are equal to 1%.
- The time value of money means that 1 dollar today is worth more than 1 dollar tomorrow because of the interest that it can earn.
- Returns must not be averaged, but compounded over time.
- Interest rate quotes are *not* interest rates. For example, stated annual rates are usually not the effective annual rates that your money will earn in the bank. If in doubt, ask!
- The discounted present value (PV) translates future cash values into present cash values. The net present value (NPV) is the sum of all present values of a project, including the investment cost (usually, a negative upfront cash flow today).
- The values of bonds and interest rates move in opposite directions. A sudden increase in the prevailing economy-wide interest rate decreases the present value of a bond's future payouts and therefore decreases today's price of the bond. Conversely, a sudden decrease in the prevailing economy-wide interest rate increases the present value of a bond's future payouts and therefore increases today's price of the bond.
- The NPV formula can be written as

  (Omitted eq)

  In this context, r is called the discount rate or cost of capital, and $1/(1+r)$ is called the discount factor.
- The net present value capital budgeting rule states that you should accept projects with a positive NPV and reject projects with a negative NPV.
- In a perfect market, firms are worth the present value of their assets. Whether firms grow quickly or slowly does not make them more or less attractive investments in a perfect market, because their prices always already reflect the present value of future cash flows.
- In a perfect market, the gains from sudden surprises accrue to old owners, not new capital providers, because old owners have no reason to want to share the spoils.