

Perpetuities

(Welch, Chapter 03-A)

Ivo Welch

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1: Maintained Assumptions

- ▶ We assume **perfect markets**:
 1. No differences in opinion.
 2. No taxes.
 3. No transaction costs.
 4. No big sellers/buyers—infinately many clones that can buy or sell.
- ▶ We again assume **perfect certainty**, so we know what the RoR is on every project.
- ▶ We assume constant RoRs (per year).

2: General Questions

- ▶ Are there any shortcut NPV formulas for long-term projects—at least under certain common assumptions?
- ▶ Or, do we always have to compute long summations for projects with many, many periods?
- ▶ Why do some of the folks have the magic ability to quickly tell you estimates that would take you hours to figure out with the NPV formula?

3: Specific Sample Questions

- ▶ What is the value of a firm that generates \$1 million in earnings per year and grows by the inflation rate?
- ▶ If your firm earns \$5 million/year, and the interest rate is 5%, what is its approximate value?
- ▶ What is a Pro-Forma terminal market-value estimate?

4: Simple Perpetuities

A **Perpetuity** is a financial instrument that pays C dollars per period *forever*.

- ▶ If the interest rate is constant and the first payment from the perpetuity arrives in period 1,

$$PV(C,r) = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t} = \frac{C}{r} .$$

- ▶ This notation is *very common* in finance:
 - ▶ C and r are the two real input variables.
 - ▶ t is an ephemeral counter (not an input variable).

5: Perpetuity Footnotes

**Make sure you know when the first cash flow begins:
Tomorrow [t=1], not today [t=0]!**

- ▶ I sometimes write C_1/r to remind myself of timing, even though cash flows are the same at time 1 as they are at time 25—I could have written C_{25} instead.

6: (NFL) Booth Review

Write out the formula $\sum_{t=1}^{\infty} \frac{C}{(1+r)^t}$:

7: Programming Language

$\sum_{t=1}^T f(t)$ is

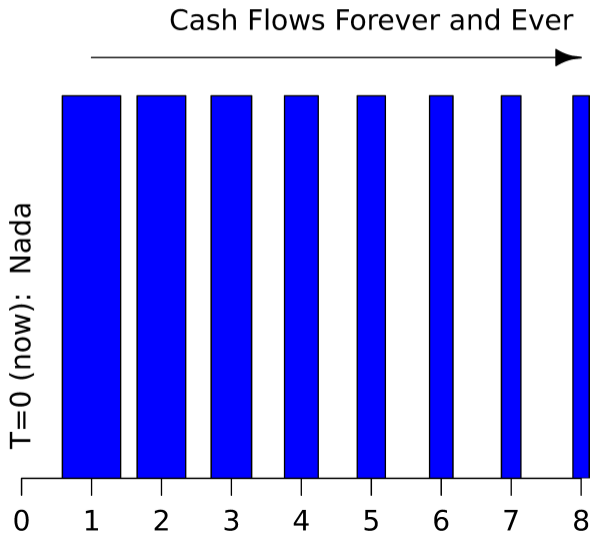
```
function sum(integer T)
  sumup <- 0.0
  for t from 1 to T
    sumup <- sumup + f(T)
  end
  return sumup
end
```


8: Infinite Sums?

How can an infinite sum be worth less than infinite cash?

- ▶ Because each future C is worth *a lot* less than the preceding C .
- ▶ In the graph on the next page, the PV of each cash flow is the bar's area.
- ▶ Soon, terms add almost nothing.

9: Graph: Perpetuity



10: Value of Perpetuity I

What is the value of an unbreakable promise to receive \$10 forever, beginning next year, if the interest rate is 5% per year?

11: Value of Perpetuity II

What is the value of an unbreakable promise to receive \$10 forever, beginning **this** year, if the interest rate is 5% per year?

12: Perpetuity Formula Mod

What is the perpetuity formula if the first cash flow starts today rather than tomorrow?

13: Nerd: Time Consistency

- ▶ Assume an interest rate of 10%.
- ▶ A perpetuity today with \$1 forever is worth $\$1/0.1 = \10 .
- ▶ A perpetuity tomorrow with \$1 forever will be worth $\$1/0.1 = \10 tomorrow.
- ▶ Today's perpetuity gives you \$1 extra next period, and leaves you with a then \$10 perpetuity. At 10%, they are worth $\$1/(1+10\%)$ and $\$10/(1+10\%)$, respectively. The latter is next year's perpetuity.

14: Growing Perpetuities

A growing perpetuity pays

- ▶ C next year
- ▶ then $C \cdot (1 + g)$ the following year,
- ▶ then $C \cdot (1 + g)^2$ the following year,
- ▶ then ...

Growing perpetuities generalize simple perpetuities ($g = 0$).

15: Growing Perpetuity Table of Cash Flow and Present Values, $g=10\%$

Time	Cash Flow	Is Worth Today
0	\$0	\$0
1	\$100	\$100
2	$\$100 * 1.1$	\$110
3	$\$100 * 1.1^2$	\$121
4	$\$100 * 1.1^3$	\$133
...
t	$\$100 * 1.1^t$...
...

16: Growing Perpetuities Formula

The PV of a growing perpetuity is

$$PV(C_1, g, r) = \sum_{t=1}^{\infty} \frac{C_1 \cdot (1+g)^{t-1}}{(1+r)^t} .$$

The real beauty is the shortcut formula,

$$PV(C_1, g, r) = \frac{C_1}{r-g} .$$

17: Growing Perpetuities Footnotes

You must memorize the shortcut formula, and know what it means!

- ▶ The growth term g acts like a reduction in the interest rate r .
- ▶ The time subscript for the payment matters now, because $C_1 \neq C_2 \neq C_t$.

18: (NFL) Booth Review

Check the growing perpetuity formula by hand!

19: Infinite Sums?

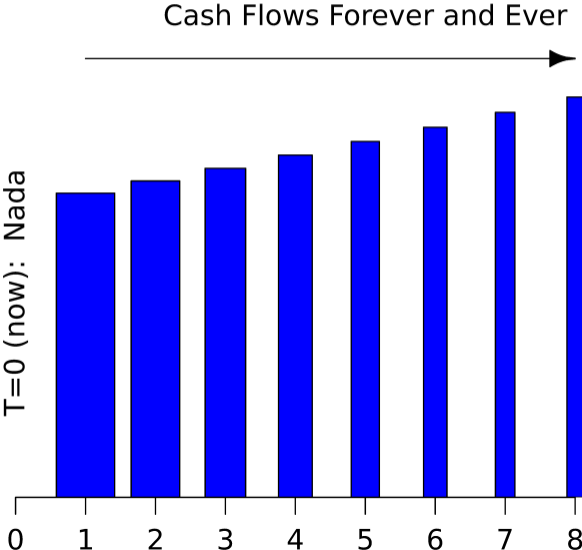
How can an infinite sum be worth less than infinite cash?

- ▶ Because the growth g is not too fast.
- ▶ Each rectangle is smaller than the preceding one, i.e., each PV is smaller than the preceding one.

What if $g \geq r$?

- ▶ The formula then makes no sense.

20: Graph: Growing Perpetuity



21: Value of Eternal Guarantee

What is the value of a guarantee to receive \$10 next year, growing by 2%/year (just the inflation rate) forever, if the interest rate is 6%/year?

22: Growing Formula Mod

What is the value of a firm that just paid \$10 **this** year, growing by 2%/year forever, if the interest rate is 5%/year?

23: Example PV Calc I

What is the formula for the value of a firm which will only grow at the inflation rate, and which will have \$1 million of earnings next year?

24: Example PV Calc II

- ▶ In 10 years, a firm will have annual cash flows of \$100 million.
- ▶ Thereafter, its cash flows will grow at the inflation rate of 3%.
- ▶ If the applicable interest rate is 8%, estimate its value if you will sell the firm in 10 years?
- ▶ What would this “terminal value” be worth today?

25: *Pro Forma* TVE

Terminal Value Estimates are the most common use of the formula:

- ▶ guesstimate the PV of the firm after an arbitrary T years in the future.
- ▶ The inflation rate is often the common long-run growth rate, g .
- ▶ A typical T in a “pro-forma” would be 5-10 years.

26: Gordon Dividend Growth Model

What should be the share price of a firm that

- ▶ pays dividends of \$1/year,
- ▶ whose dividends grow by 4% every year, and
- ▶ which will continue to do so forever,
- ▶ if its cost of capital (CoC) is 12%/year?

27: GDGM for CFAs

CFA Exam: Using D for C gives you the GDGM (Gordon Dividend Growth Model):

$$P = \frac{D}{r - g} .$$

Ergo $D/P = r - g$.

28: GDGM for Real

Don't trust the GDGM

- ▶ Firms can shift dividends!
- ▶ What a firm does not pay out in dividends today will make more hey (dividends) tomorrow.
- ▶ It should not matter if the firm cancels its \$1 dividends this year in order to pay out an extra \$1.05 next year.

29: GDGM Improvements?

- ▶ An improvement uses the **plowback ratio**:
 - ▶ it takes into account that reinvested cash should pay more dividends in the future,
 - ▶ but it's still just lipstick on a pig.

- ▶ A better valuation formula could use earnings instead of dividends,
 - ▶ because earnings are more difficult to shift around.

30: GDGM Implied Cost of Capital (ICC)

What is the CoC for a firm that

- ▶ pays a dividend yield (D/P) of 5%/year today,
- ▶ if its dividends are expected to grow at a rate of 3%/year forever?

31: GDGM ICC Formula

An *Implied Cost of Capital (ICC)* is the expected RoR embedded in the stock price today.

- ▶ GDGM is sometimes used to estimate an implied cost of capital, ICC,
- ▶ via the inverted formula $r = D/P + g$.
- ▶ A higher P today implies a lower implied CoC at which the firms can obtain capital from investors.

32: Advanced Finance

- ▶ Possible to show that a formula like the GDGM is more general
- ▶ If a firm has a high D/P , it must mean that it will either have a high r or a low g .
 - ▶ If a firm has a div yield of 50%, if this div yield will not go down, we will make a huge r .
 - ▶ If a firm has a div yield of 0%, if this div yield will not go up, we will make a lousy r .

33: S&P500 ICC

- ▶ The current P/E ratio of stocks is, e.g., at <https://www.currentmarketvaluation.com/models/price-earnings.php>.
 - ▶ In 2022, this was about 30 for the S&P500.
 - ▶ The historical average was about 20. (30 is about one sd more.)
- ▶ The growth rate of earnings is, e.g., at <https://www.multpl.com/s-p-500-earnings-growth/table/by-year>.

- ▶ If future GDP growth rate is about 3-5% (the growth rate of GDP), what should investors reasonably expect about future RoR implied by a P/E ratio of 30?

34: Quick Calc: Value of Firm

Our firm has earned \$100,000 this year.

It has stopped growing in *real* terms.

The current interest rate is 6%/year.

The inflation rate is 2%/year.

What is the value of our firm?

- ▶ What is it over-the-envelope ?
- ▶ What is it exactly?
- ▶ What is the first cash flow?

35: Growth Rate of Google

- ▶ In May 2022, Alphabet (Google)'s share price was about \$2,200. Its EPS was about \$110.
- ▶ Therefore, Google's P/E Ratio was about 20.
- ▶ Google's CoC for equity was about 6%/y.
- ▶ What does the market believe G's as-if-eternal earnings growth rate will be?

36: Metaphysics

Are perpetuities meaningful?

- ▶ How long will firms last?
- ▶ How long will Google last?
- ▶ What firms or institutions have survived from the Roman empire?