Investing Choices and Risk Measures

(Welch, Chapter 08)

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1: Maintained Assumptions

Perfect Markets

1. No differences in opinion.
2. No taxes.
3. No transaction costs.
4. No big sellers/buyers—infinitely many clones that can buy or sell.

With risk and risk aversion

2: First Question: Investors

- **How should I choose among many different projects?**
  - taking market prices as fixed and given.

3: Second Question: Corporate Managers

- How do (your) projects determine company risk?
- How do (your) investors think?
- What will make them want to give you money?
- What is *your* opportunity cost of capital $E(r)$?

4: Investor Risk Preferences

- They care only about their overall portfolio, not about your own or any one project per se.
  - Investors don't care about your or any other specific project.
  - Your own project's SD may or may not matter to them. We will have to figure this one out.
- We assume investors care about the standard deviation (SD) of their ***overall*** *portfolio* return.

5: Investments

- Four equally likely scenarios ("states"):
  - states: yellow, red, green, blue.
  - Nerdnote: "state-based" preferences are more general than our Mean/SD preferences.
- Four investment assets: A, B, C, D.
- Returns (in Percent or Dollars).

6: Investment Contingencies

	Ylw	Red	Grn	Blu
A:	−4.0	−4.0	+6.0	+6.0
B:	−1.0	+9.0	+9.0	−1.0
C:	−1.25	+1.25	+3.75	+1.25
D:	+3.0	+13.0	+3.0	−7.0

7: Investment Rewards

- What are the rewards of the four investments?
  - We measure "rewardiness" as expected rate of return.
  - "Risk and reward" usually means "expected standard deviation and expected mean."
    - PS: The standard deviation is a known number today. Thus standard deviation and *expected* standard deviation and *exp. exp.* standard deviation are all the same thing.

8: Investment Risks

- What are the risks of the four investments?

9: Population vs Sample Statistics

- If Ylw-Blu returns are just representative historical realizations, you would divide by 3, not 4 in your computation of the variance.
- In real life, we rarely have population statistics.
  - Historical data are sadly often our best choice,
  - but *here*, for learning, we assume we know the distribution.

10: Overall vs Parts Risk

The standard deviation is a meaningful measure of risk only for your overall portfolio.

We need not care about the standard deviations of individual investments.

11: Means and SDs of 4 Assets

	Mean	X-Mean	Var	SD
A:				
B:				
C:				
D:				

12: Portfolio Risk

- What is the risk of an (equal-weighted) portfolio of Asset A and Asset B?
  - (HINT: First compute the RoRs of the combination portfolio in each state.)

13: Portfolio Risk

- Is the average portfolio or are the individual components riskier?

- Why?

14: Good Portfolios?

- What kind of portfolio would you — *a smart but risk-averse investor* — hold?

15: Real Life Prime Portfolio

- In real life, what portfolios should and do smart investors with risk-aversion hold?

16: Portfolio Risk

- What is A's portfolio risk if you add C to your portfolio vs if you add D to your portfolio?

	Mean	X-Mean	Var	SD
A+C:				
B+D:				

17: Riskier Investment I

- Is C or D the riskier investment in itself?

18: Riskier Investment II

- If you already own A, is C or D the riskier addition?

19: Why?

20: Base Portfolio

- If investors are smart, what is their base portfolio A?

21: CAPM Preview

- **Previewed Guess**: If you are selling to smart investors either C or D, for which of these two projects do you think will investors clamor to invest in your project?
  - More clamoring : they will accept lower exp. RoR.
  - Lower exp. RoR : they will pay a higher price.

22: Fundamental Investment Insight

- Investors (should) care about overall portfolio risk, not about the constituent component risk.
- From a corporate managerial perspective, it is not low-risk projects that investors like, **BUT** projects which wiggle opposite to the rest of their portfolios.

23: Synchronicity

- How should we measure synchronicity?
  - For exposition, consider A to be the market portfolio that investors are already holding.
  - We need a measure of how synchronous or non-synchronous any new stock/asset/project is with respect to this portfolio A=M.

24: Calculate Ri-Mean(Ri)

| | Ylw | Red | Grn | Blu | |—-+——-+——-+——+——-+ | A: | −5.0 | −5.0 | +5.0 | +5.0 | | B: | −5.0 | +5.0 | +5.0 | −5.0 | | C: | −2.5 | 0.0 | +2.5 | 0.0 | | D: | 0.0 | +10.0 | 0.0 | −10.0 |

25: Calculate COVAR

- Covariance is mean of cross-products:

$$(A-\overline{A})\times(B-\overline{B}) = +25,-25,+25,-25.$$

$$(A-\overline{A})\times(C-\overline{C}) = +12.5,0,+12.5,0.$$

$$(A-\overline{A})\times(D-\overline{D}) = +0,-50,0,-50.$$

$$cov(A,B) = 0, \quad cov(A,C) = +6.25, \quad cov(A,D) = -25.$$

- why are we demeaning and multiplying?

26: Calculate Beta (Slope)

- Beta is the covariance divided by the variance:

$$\beta_{C,A} = 6.25/25 = 0.25\ ,$$

$$C = 1 + 0.25 \cdot A, \qquad (A = -1.5 + 2 \cdot C)\ .$$

- Ergo, beta of C (y) on A (x) is 0.25. "*A-beta of C*"

$$\beta_{D,A} = -25/25 = -1,$$

$$D = 4 - 1 \cdot A, \qquad (A = 2.5 - 0.5 \cdot D)\ .$$

27: Calculate Correlation

- The correlation is the covariance divided by the standard deviations of its two ingredients:

$$cor(A,C) \approx 0.7071 \qquad cor(A,D) \approx -0.7071 \ .$$

- The order does not matter for covariance or correlation. It matters only for beta.

28: Risk Contribution Measures?

- Covariance generalizes variance. (Why?) Thus, it also has uninterpretable units. Yuck.

- Correlation has a scale problem:
  - Think you own one investment paying -1% or +1%.
  - An investment with -0.01% or +0.01% has perfect corr (1.0).
  - An investment with -100.0% or +100.0% has perfect corr (1.0).
  - Which one would contribute more risk?

29: Best Measure: Market-Beta

- The best risk contribution of adding B to M is B's market-beta with respect to M.
  - This means *var*(*Rm*) is the denominator.
  - Without verbal qualification, beta always means with respect to Rm, i.e., market-beta.
  - Market-beta is the "M-beta" of asset B.

30: Happy Family

- Covariance, correlation, and beta have the same sign.
- They differ only in magnitude.

31: Beta is a Line Slope

- Beta is a slope. Put A (M) on the X axis, and your project B (or C) on the Y axis.
  - A slope of 1 is a diagonal line.
  - A slope of 0 is a horizontal line.
  - A slope of ∞ is a vertical line.
- Without alpha, beta tells you how an x% higher RoR (than normal) in the market will likely associate with simultaneously higher RoR (than normal) in your stock.
  - think $\beta_i \cdot x\%$ higher rate of return (than normal)

32: Beta Interpretation

- The stock-return beta helps with a conditional forecast of Ri, given Rm.
  - If you can go short the market, you can even (on average) reduce your portfolio's net market exposure down to 0. Beta tells you how short you should be.

33: Beta Measurement

- Mediocre measures of market-beta are available on every finance website.
  - A better estimator would be to use daily stock returns on 1-2 years of historical data.
  - The best market-beta measure shrinks smartly.
    - winsorizing means trimming to limits.
  - Do not use other stocks' betas if you do not have to.

34: Market-Beta of Market

- What is the market-beta of the overall stock market (say, the S&P500)?

35: Market-Beta of Risk-Free Rate

- What is the market-beta of the risk-free rate?

36: High vs Low *Beta* Projects

- Given equal expected returns, what's more desirable to an investor heavily loaded up on the stock market?
  - A project with a high beta? Or
  - A project with a low beta?

37: High vs Low *Risk* Projects

- Should high or low variance projects have to offer higher expected RoRs?

38: High vs Low *Beta* Projects

- Should high or low beta projects have to offer higher expected RoRs?

39: Conglomeration

- New Firm: 40% C and 60% D. ($4m and $6m.)
- What is the average RoR (mean)?
- What is the average variance?
- What is the average sd?
- What is the average beta?

40: Value-Averaging

- Which statistics can you “value-average”?
- Which statistics can you not “value-average.”

41: Corporate Market Beta

- Is there a quicker way to compute the overall market-beta of your firm, based on the market-betas of its constituent projects?

42: Nerd Warning: Time-Changing Pfio Weights

- Firms in portfolios constantly change their relative investment weights:
  - In historical data use, the weighted formula requires fixed weights.
  - Because more appreciating stocks get more weight, this often means that you cannot use today's investment weights in a historical calculation.
  - Put differently, your portfolio weights and overall beta can change all the time.

43: Mean-Variance Frontier

(Omitted, but some discussion in book.)

- The mean-variance efficient frontier (= the mean-standard deviation efficient frontier).
  - optimal combination of assets.
  - covering it would require 2+ full lectures.
  - Underlies CAPM+. Take an investments course!
  - It is in common (practical) use.
  - Important.

44: Variance of Weighted Sum

If pfio P consists of two assets, A and B:

$$r_P = w_A \cdot r_A + w_B \cdot r_B \ ,$$

then the portfolio variance is

$$Var(r_P) = Var(w_A \cdot r_A + w_B \cdot r_B) =$$

$$w_A^2 \cdot Var(r_A) + w_B^2 \cdot Var(r_B) +$$

$$2 \cdot w_A \cdot w_B \cdot Cov(r_A, r_B) \ .$$

- This "$2 \cdot w_A \cdot w_B \cdot Cov(r_A, r_B)$" term means it is *not* the weighted average, "$w_A \cdot Var(R_A) + w_B \cdot Var(R_B)$"!

- One cannot value-weight variances!
  - A portfolio of half A and half B usually does not have a variance that is the average of the variances of A and B.

45: Effect of Changing Weights

- The generalized variance formula is based on variance-covariance matrix between all assets and all your investment weights.
- It makes it easier to recompute the portfolio risk when you change portfolio weights.

46: Time Correlation

- What is the expected correlation of stocks' RoRs from one day to the next day?

47: Time Correlation

- Can the expected correlation be *a* little off from 0?

48: Time Correlation

- Can the *actual* realized correlation be a little off from 0?

49: Time-Adjusting Risk

- This is an important application of the variance-covariance formula.
- If the risk of investing in x for 1 year is σ=20%, what is the risk of investing for 10 years?

50: A1: Constant Risk

- Let's assume that the per-unit-of-time standard deviation remains constant.
  - This need not be the case.
    - It is an additional assumption.
    - Risk before harvest may be different than risk before planting.
  - We can then omit time subscript.
  - We call this constant annual number σ.

51: A2: Uncorrelated over Time I

- Rates of return over time should be uncorrelated
  - This is a consequence of a perfect market.
  - Otherwise, you could use past stock returns to outpredict future stock returns.
  - If not exactly zero, likely just a little statistical noise.
- Algebraically, for $s \neq 0$,

$$Cov(R_t, R_{t+s}) \approx 0 \ ,$$

where the subscripts t and $t + s$ refer to two time periods, not to different stocks at the same time.

52: A2: Uncorrelated over Time II

- In this case, the following approximation is not bad:

$$\boxed{Sd(R_{0,T}) \approx \sqrt{T} \cdot \sigma}$$

- **Example**: if your portfolio risk is 10% per month, then your annual risk is about $\sqrt{12} \cdot 10\% \approx 35\%$ per year.

53: Time-Adjusted Derivation

$$Var(R_{0,T}) \approx Var(R_{0,1} + R_{1,2} + \ldots + R_{T-1,T})$$

$$= Var(R_{0,1}) + Var(R_{1,2}) + \ldots + Var(R_{T-1,T})$$

$$= \text{many 0 covariance terms}$$

$$\approx T \cdot \sigma \ .$$

54: Sharpe Ratio

- **Sharpe-Ratio (SR)**: *a* (badly flawed but common) measure of investment performance:
  - $$\mathrm{SR}_i = \frac{\mathrm{Avg}(R_i - Rf)}{SD(R_i - Rf)} \approx \frac{Avg(R_i) - Rf}{SD(R_i)} \ .$$
  - The SR grows with the square-root of time.
  - Calculated typically from *monthly* RoRs ***annualized*** by $\sqrt{12}$.
  - Historical SR of Stock Market: $4\%/10\% \approx 0.4$.

55: Nerd: VW/EW Portfolio Maintenance

- Is it easier to maintain a value-weighted or an equal-weighted portfolio?