Quotes vs. Rates, NPV, and Basic Capital Budgeting

(Welch, Chapter 02)

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Quote vs Rate

If your bank pays you 50% per year, what is your RoR after 2 years?

Portfolio RoR

You have \$100. You invest half each in two firms.

- Firm 1 makes 10% this month.
- Firm 2 makes 20% this month.

How much did your portfolio make in total?

Hint: $\$(1.20 \cdot 1.1) = \$1.31$.

The Cross-Term

Can you guess what people mean by the "cross-term" in the compounding formula?

- The difference between adding rates $(r_{0,1} + r_{1,2})$ and compounding returns $[(1 + r_{0,1}) \times (1 + r_{1,2}) - 1]$ is the term $r_{0,1} \times r_{1,2}$.
- It is the *interest on the interest*.
- It is often small *in the short run*.

Interest Rate Compounding I

If the 1-month interest rate is 1%, what is the 1-year rate?

Interest Rate Compounding II

If the 1-day rate is 0.02%, what is the 7-day (weekly) rate?

Compounding Approximation

How good an approximation is simply adding RoRs?

- This depends on the crossproduct. It may or may not be worth worrying about.
- This will be covered again below.

Logarithm Algebra Reminders

$$x^a = b \Leftrightarrow x = b^{1/a} \ ,$$

and

$$a^x = b \Leftrightarrow x = \frac{\log(b)}{\log(a)} \ .$$

Interest Rate (De-)Compounding

A project for $200 promises to return 8% per year, but allowing fair withdrawals at any time.

How much money will you have after one month?

Annual Rate to Daily

If the annual interest rate is 14%, once compounded, what is the daily rate?

Monthly Rate to Weekly

The monthly interest rate is 1.5%. There are 30.4 days in the average month. What is the weekly rate? Are there different ways to calculate it?

Doubling Return

If you are doubling your money every 12 years, what is your RoR per year?

Doubling Time

If *a* project promises to return 8% per year, how long will it take for you to double your money?

Compounding

Is compounding more like "*adding*" rates or "*averaging*" rates?

Approximation

If both the interest rate and the number of time periods is small:

$$(1 + r_n)^t \approx 1 + t \cdot r_n \ .$$

Adding up instead of compounding becomes a worse approximation when time and interest rates increase.
(It also matters how much money is at stake.)

Warning: Conventions and Jargon

- Interest rates (and quotes) are tedious and often confusing because everyone computes and quotes them slightly differently.
- Sometimes, interest rates are intentionally obscure in order to deceive the unwary..
- Always know what you are talking about and ask if you are unclear.

Bank *Quote* Example

A bank quotes you 8% interest per year. If you invest $1 million in the bank, what will you end up with?

Interest Quotes (Not Rates)

Many institutions give you interest “quotes,” rather than interest rates, and the two are easy to confuse.

- This is especially bad with annualized interest quotes. There are many “pseudo interest rates” which are really “interest quotes” and not true “interest rates.”

Bank Interest Quotes (Not Rates)

Banks and lenders sometimes calculate and pay daily interest rates, although they only credit the interest payment to accounts once per month

- Such banks' daily interest rate calculation is the quoted annual interest rate divided by 365 ($r_d = r_y/365$) (Note: some banks use 360 days)

Some banks quote for clarity

- Interest rate: 8% compounded daily
- Effective annual yield: 8.33%

Example Bank Rate

Is the 8% posted by the bank *a* true annual interest rate?

Bank Interest Rate

Is the ”effective annual yield” of 8.33% *a* true annual interest rate?

Quotes vs Rates: Treasuries

At auction, the Fed sells T-bills that pay \$10,000 in 180 days. If the discount quote is 10% (which is absolutely *not* an interest rate), you can purchase one for \$9,500. Fed quotes:

$$\$100 \cdot \left[100 - \left(\frac{\text{days to maturity}}{360}\right) \cdot \text{discquote}\right] \;.$$

$$= \$100 \cdot [100 - (180/360) \cdot 10] = \$9{,}500 \;.$$

- Note: Financial websites often print "95" instead of "9,500," because it is shorter, and T-bills are quoted in units of 100.

Treasury Quotes

- Do not memorize the Treasury formula.

Assume the Treasury quote is indeed 95. If you invest \$1, how much will you receive in 6 months? 95%?

Present Value Introduction

Arguably, **Present Value** is the most important concept in (corporate) finance.

Present Value Example

If you will receive $7 next year, and the prevailing interest rate (= [opportunity] *cost of capital*) for investing is 40%, what do you value this “$7 next year” as of today?

What PV Formula?

And how does PV relate to the basic RoR formula?

PV 2Y Application

If you will receive $7 in two years, and the prevailing (alternative) interest rate [or cost of capital] is 40%/year, what do you value this $7 as of today?

The General PV Formula

The present value of cash CF_t at time t is translated into today's equivalent via

$$PV = CF_0 = \frac{CF_t}{(1 + r_{0,t})} .$$

- **Discount factor:** $1/(1 + r_{0,t})$
  - The df multiplies a future CF in order to obtain its PV
- **Discount rate:** $r_{0,t}$
  - This the key input into the df.

Ashes to Ashes, Oranges to Oranges

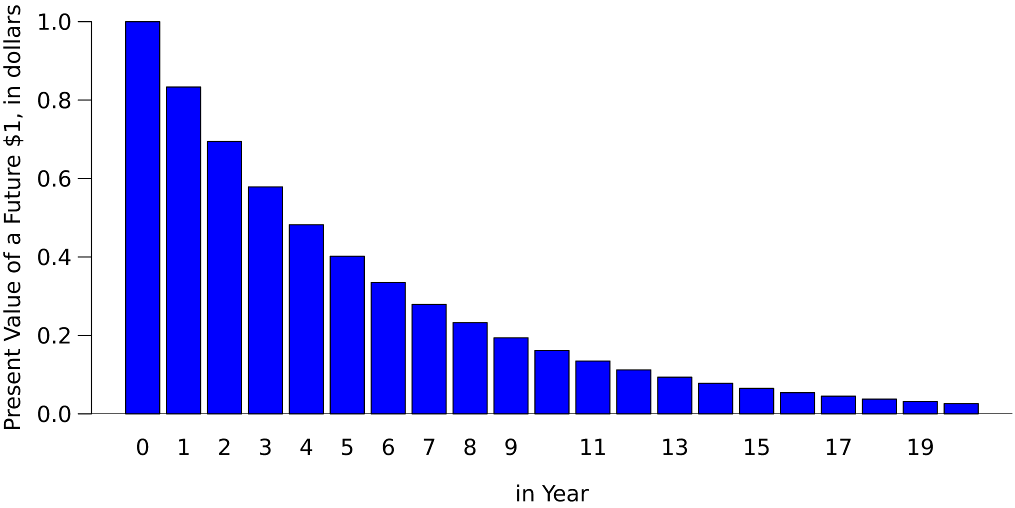
Think of PV and NPV as first converting all oranges (differently timed future cash flows) into apples (cash flows as if they were right now), so that you can compare cash flows—apples to apples and not apples to oranges.

PV Formula Jargon

The discount rate is often called the **(opportunity) cost of capital**

- Think of the CoC as either your alternative investment opportunities' return (if you have money) or your cost of borrowing (if you need money).
- In our PCM, the two are the same. You can invest into or borrow from infinitely many alternatives for the same RoR = CoC.

Graph: Discounting at 20%/Y



Net Present Value (NPV)

NPV simply means include the time-0 cash flow, often *a* cost (negative).

Interest Rates & Bond Prices

How does the price of *a* bond change if the economy-wide interest rate changes (up)?

Multiple-Year PV Example

If project ABC will pay $7 next year and another $7 in two years, and the prevailing CoC is 40% per year, would you value this project at $14?

What formula are you using?

Capital Budgeting

- **Capital Budgeting** is an anachronistic but ubiquitous term. It implies that you have a fixed capital budget that allows you to take a limited number of projects.
- A **capital-budgeting rule (CBR)** gives a “take it or leave it” answer.

NPV Capital Budgeting I

If ABC costs $8, should you take it or leave it?

NPV Capital Budgeting II

If the CoC were 80%/year instead, should you take this project?

NPV Formula

The net present value (NPV) is

$$\text{NPV} = CF_0 + \frac{CF_1}{(1+r_{0,1})} + \frac{CF_2}{(1+r_{0,2})} + \dots$$

$$= \sum_{t=0}^{\infty} \frac{CF_t}{(1+r_{0,t})} \; .$$

It is called “net” because the CF_0 is often negative.

NPV Capital Budgeting Rule

Already stated. The **NPV capital budgeting rule** (CBR) is

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**TAKE ALL POSITIVE NPV PROJECTS**.
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- The NPV rule is correct and optimal under PCM and certainty.
  - Other rules *may* leave money on the table

Proof by Arbitrage

1. Borrow and take the positive NPV project.
2. Pocket the difference.
3. Do this infinitely many times (arbitrage).
4. Use capital markets to shift income to whenever you want to consume.

Scarcity?

Q: What if someone just invented a device that makes everything better?

A: The equilibrium CoC will go up.

Positive NPV projects must be scarce. Otherwise, money would compete to bid up r.

Good/Bad Investments?

Is a good firm (or stock) a good investment?

- A good firm could mean a growing business.

Is a bad firm (or stock) a bad investment?

- A bad firm could mean a shrinking business.

Fast vs Slow Growing Firms

- In a PCM, neither is a better investment, because both should be priced fairly.
- In the real world, the question is whether the price is too high or too low.
  - if the price is too high, either can be a bad investment.
  - if the price is too low, either can be a good investment.
- If stupid investors think growing firms are better and drive the price too high (by piling in),
  - this would make the shrinking firm the better investment.