

Uncertainty, Default, and Risk

(Welch, Chapter 06-A)

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Mon Jun 20 23:48:30 2022

Maintained Assumptions

Perfect Markets

1. No differences in opinion.
2. No taxes.
3. No transaction costs.
4. No big sellers/buyers—infininitely many clones that can buy or sell.

BUT NO LONGER Certainty

Sadistics

- ▶ We now need to predict (describe) the future.
- ▶ For this, we need statistics.

Legal Disclaimer

This chapter may be illegal in some states. Attend the rest of this course at your own risk.

For example:

Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of \$ 250 and/or six months in prison.

(Section 889, New York State Code of Criminal Procedure.)

Statistics

- ▶ Covered more fully in your statistics course.
- ▶ This brisk overview is just the minimum needed.

Random Variables (RVs)

- ▶ A **Random Variable (RV)** (often with tilde over the variable) is
 - ▶ not an ordinary algebra variable,
 - ▶ that has some specific unknown value,
 - ▶ but best thought-of as a histogram with a name,
 - ▶ a “cloud” outcome from a randomizing device (e.g., a coin or a card draw), whose histogram defines it.

Inference

- ▶ In many applications, statisticians *assume* you understand the system and know the histogram, but not the draw (the outcome).
 - ▶ **Great for:** coin, die, roulette.
 - ▶ **Not sure for:** financial markets (where we do not understand the underlying physics).
- ▶ On occasion, I will warn you.

Die Throw RV

- ▶ Our main random variable example will be the payoff you get after a die is thrown:
 - ▶ 1 = -\$6,
 - ▶ 2 = +\$36,
 - ▶ 3 = -\$12,
 - ▶ 4 to 6 = +\$150.

Let's call this random variable D (ie \tilde{D}).

- ▶ Draw its histogram.

Draw the Histogram of D

The Expected Value

- ▶ $E(\cdot)$ is common notation for *Expected Value* (EV).
 - ▶ Example: $E(D)$ is the expected value of D .
- ▶ You can calculate the EV from the histogram:
 1. multiply each outcome by its probability,
 2. and sum up the terms.
- ▶ If all outcomes are equally likely, take the average.

- ▶ Think of the expected value as **the mean of the RV if infinitely repeatedly drawn**.
- ▶ *Average, Mean, and Expected* are often used interchangeably, even if not fully correct.
 - ▶ *Average* or *Mean* is for a specific (historical?) sample.
 - ▶ *Expected* is not known yet.
 - ▶ *Average* will be a realization of *Expected*.
 - ▶ On average, expect the *Average* to be the *Expected*.

Calculate $E(RV)$

What is the expected payoff of D ?

Question

Is $E(D)$ a number or a RV?

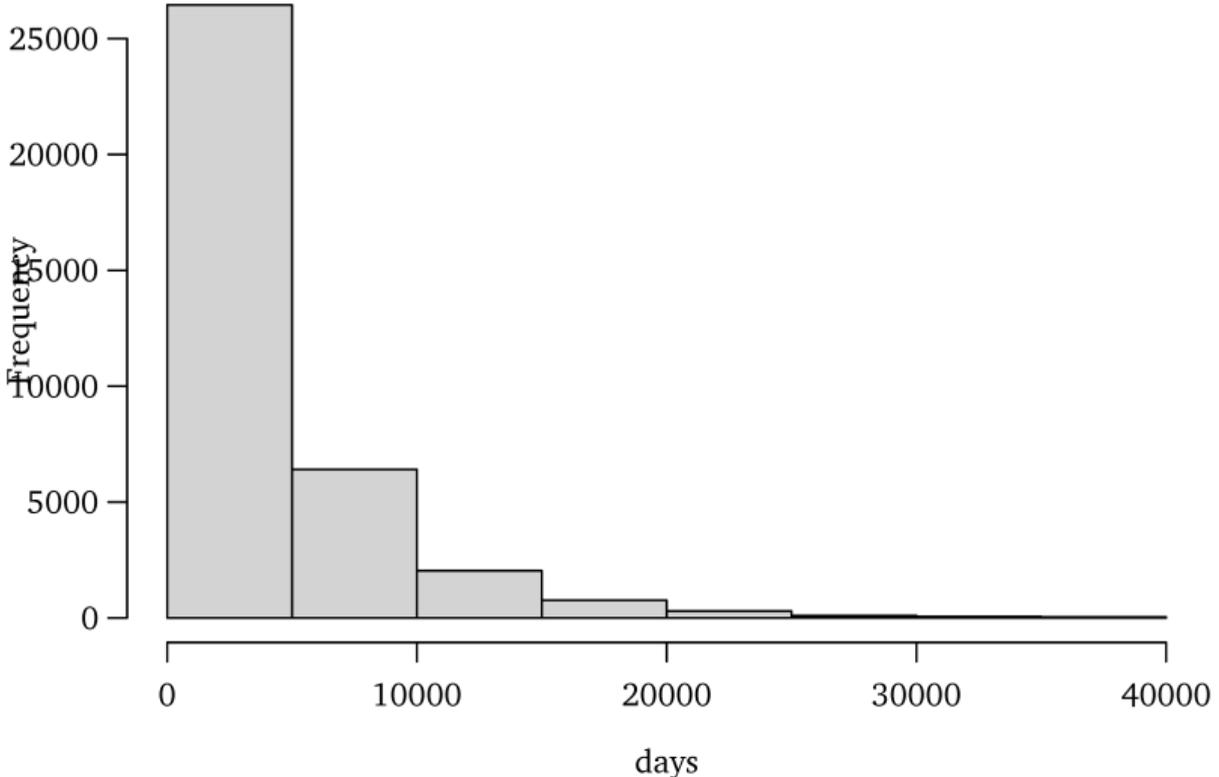
Is the Mean Mean?

- ▶ Is the expected payoff always the most likely outcome?
- ▶ Are half of all outcomes always below the mean?
- ▶ Can you have the mean outcome?
 - ▶ What is the average number of children per woman?

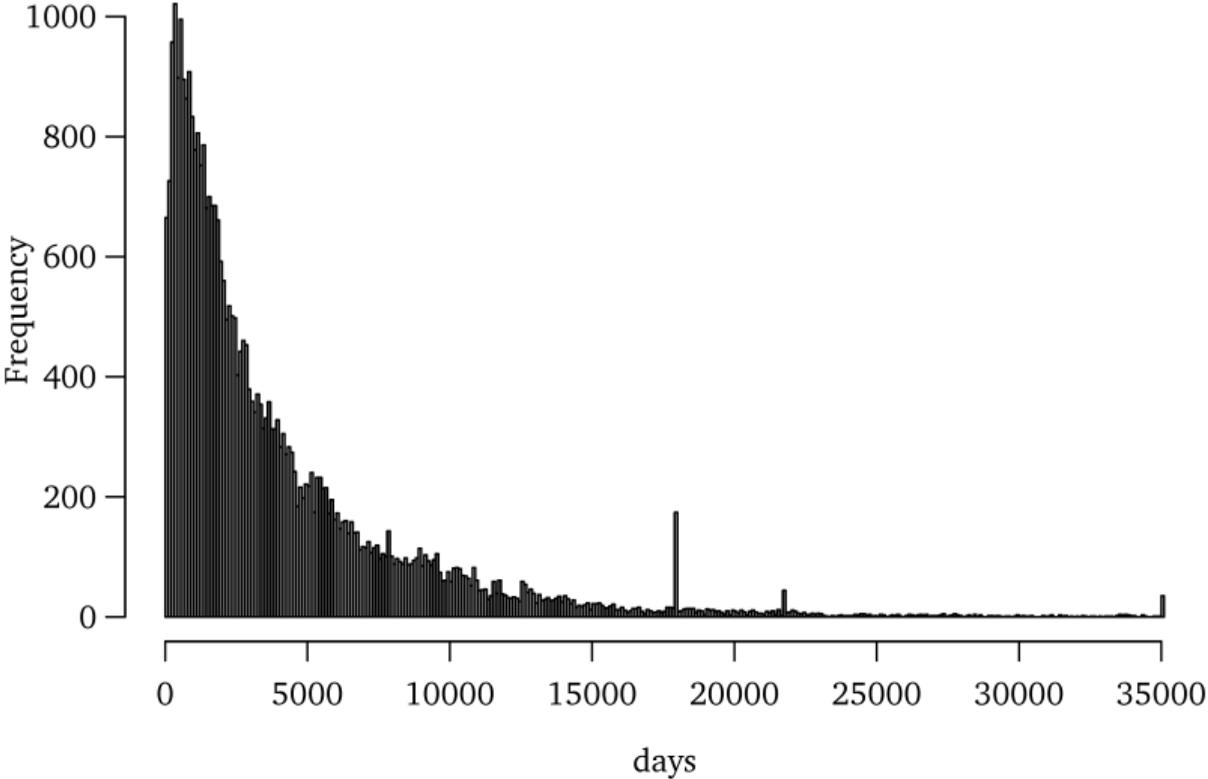
Central Statistics

- ▶ How Long Were Stocks Traded on US Public Exchanges?

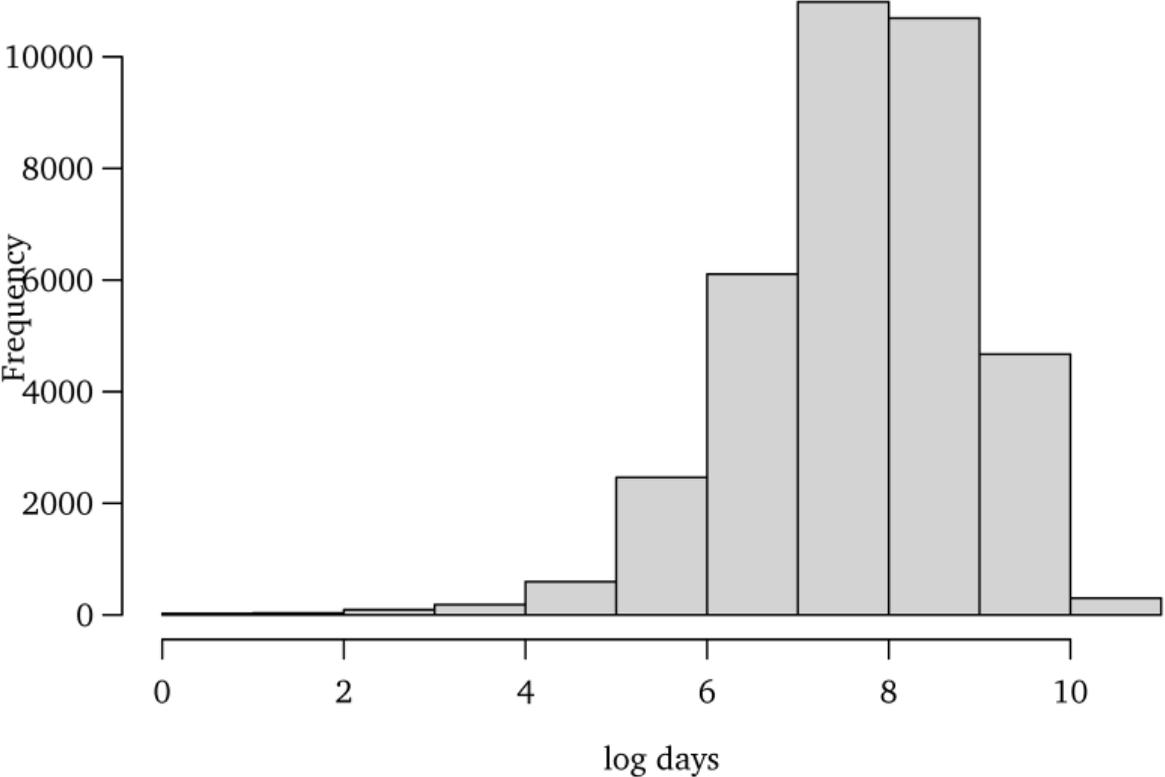
View 1: Mean = 3,992 days



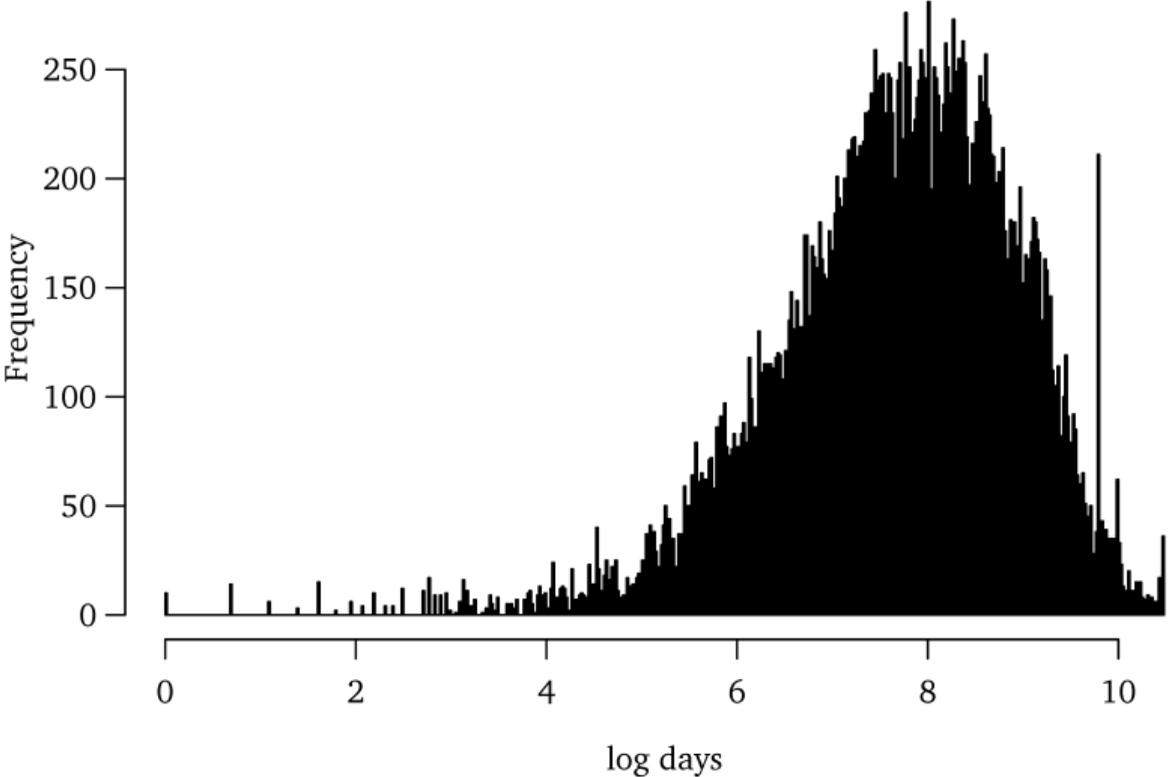
View 2: Mean = 3,992 days



View 3: Mean of Log Days = 7.679 days



View 4: Mean of Log Days = 7.679 days



WTH?

- ▶ The mean of *days* is 3,992d.
- ▶ The mean of $\log(\textit{days} + 1)$ is 7.679.
- ▶ But $\exp(7.679)$ is equal to 2,162 days.
- ▶ **NOTE:** $\text{mean}(\log(\textit{days})) \neq \log(\text{mean}(\textit{days}))?$

Warning

- ▶ Histograms can be shaped quite differently, depending on the breaks

Different Central (Plus) Statistics

- ▶ What is the mean?
- ▶ What is the median?
- ▶ What is the mode?

- ▶ (What is the standard deviation?)

Different Central Statistics

- ▶ Mean: 3,992 days (10.9 years)
- ▶ Median: 2,425 days (6.6 years)
- ▶ Mode: 17,914 days (49 years)
 - ▶ why this high a number?

- ▶ Other Statistics:
 - ▶ SD= 4,515 (12.4y)
 - ▶ IQR= 1,037 – 5,308 (50-50 in vs out)
 - ▶ Min= 0. Max=35,064 (96 years).

Functions of RVs (Like $\text{Log}(X)$)

Back to our Die:

- ▶ The $E(D)$ was \$78.
- ▶ What is the expected value of the die-squared, $E(D^2)$?
- ▶ Recall D
 - ▶ 1 = -\$6,
 - ▶ 2 = +\$36,
 - ▶ 3 = -\$12,
 - ▶ 4 to 6 = +\$150.

Linear and Non-Linear Functions of RVs

Is $E[D^2]$ the same as $[E(D)]^2$?

PS: Is $2 \times E(D) + 1 = E(2 \times D + 1)$

Measuring Unexpected?

What is an unexpected quantity?

Fair Bets

- ▶ What is a fair bet?
- ▶ What would it take for the above die-throw to become a fair bet?

Variance

- ▶ The variance is $Var(D) = E\{[D - E(D)]^2\}$
- ▶ Roughly speaking, the variance is the **expected squared deviation from the mean.**
 - ▶ This is just pseudo-intuitive.
 - ▶ Infinitely many draws: *Mean of $(D - Mean(D))^2$.*
 - ▶ the sample *turns into* the population variance.
 - ▶ The units on and meanings of variances are usually incomprehensible.
 - ▶ except that higher is more volatile
 - ▶ PS: Variance of Number of Days on CRSP = 20,385,225

Variance Example

Remember: $E(D) = \$78$. From the RV's histogram, to obtain the variance: multiply each squared deviation from the mean by its probability, and add up terms.

State	Prob	Outcome	Oc-Mn	Squared
"1"	1/6	-\$6	-\$84	\$\$7,056
"2"	1/6	+\$36	-\$42	\$\$1,764
"3"	1/6	-\$12	-\$90	\$\$8,100
"4"- "6"	3/6	+\$150	+\$73	\$\$5,184
WMean:		+\$78	\$0	\$\$5,412

Standard Deviation (SD)

- ▶ The square-root of the variance:

$$SD = \sqrt{\$5,412} \approx \$73.57 .$$

- ▶ Think of the SD as the **typical deviation from the mean** of the next draw.
 - ▶ Not entirely correct, but close enough.
 - ▶ SDs have meaningful units.

Variance or SD?

- ▶ If the variance is higher, then the standard deviation is higher and vice-versa.
 - ▶ So, “more risk” is both “more VAR” and “more SD.”
 - ▶ SD units are more meaningful,
 - ▶ but VAR is also commonly used.
 - ▶ Vanilla or Chocolate?

Very Nerdish: Signal Balance

- ▶ Let's say you want to combine two investment signals.
 - ▶ Think S_1 is $(-100,+100)$ and S_2 is $(-1,+1)$.
 - ▶ Your rank order would be
 - ▶ $(S_1=100, S_2=1)$: 101
 - ▶ $(S_1=100, S_2=-1)$: 99
 - ▶ $(S_1=-100, S_2=1)$: -99
 - ▶ $(S_1=-100, S_2=-1)$: -101
- ▶ Problem: All your best signals are really just S_1 . S_2 is defacto unimportant.

- ▶ Solution: normalize signals first. Otherwise, your best signals may give too much weight to the signal that has the higher mean—or, if means are the same, the signal that has the higher variance.
- ▶ Two common normalizations:
 1. the “percentile rank”
 2. the “normalized value” (net of mean, divided by standard deviation), also called a “Z-value”.

Big Leap of Faith

- ▶ What is the True Histogram?
 - ▶ Yes, we know the histogram for a die throw from physics.
 - ▶ But what is the histogram for the RoR on the stock market?
 - ▶ Therefore, we pretend that the (many) historical outcome realizations of RoRs can proxy for the true unknown histogram of RoRs.

Historical = Future?

- ▶ Then we pretend that this historical histogram applies to future RoRs, too.
- ▶ This is a “Hail Mary.”
 - ▶ But once we assume we know the histogram, we can calculate expectations and do much more!
 - ▶ Employment program for finance professors?

Future Stock Returns as RVs

- ▶ This translation of the historical outcome histogram (distribution) into the future outcome histogram (distribution) is a *heroic* assumption.
- ▶ But what else can we do?
- ▶ Anyone have a better idea?
- ▶ Always remain mindful of the problem!

Expected Stock Returns

- ▶ Historical extrapolation generally works poorly for mean returns.
 - ▶ BTC was a very profitable investment in the past.
 - ▶ Will it also be in the future?
- ▶ Investors try to learn and adapt all the time.
- ▶ Moreover, there is *survivorship bias*.

Expected Market Returns

- ▶ What is the expected RoR on the overall stock market (say, the S&P 500 or DJ 30)?
 - ▶ Is it still the same that it was historically?
 - ▶ Why?

Risk of Stock Returns

- ▶ Extrapolation works well for variances, standard deviations, and market-betas (TBD).
 - ▶ Not perfect, but well.
 - ▶ Will show evidence soon.
 - ▶ Ergo, statistics works well for risk management!

Historical Example

- ▶ What are the $E(R)$ and $SD(R)$ if you only knew four historical stock returns:
 - ▶ +10%,
 - ▶ -5%,
 - ▶ +20%,
 - ▶ +15%.

Sample Historical Calculations

- ▶ Mean: 10%
- ▶ Each Deviation: 0%, 15%, 10%, 5%
- ▶ Squared Deviations: 0, 225, 100, and 25,
- ▶ Sum Squared Deviations: 350.
 - ▶ Now take the mean?!

Population vs. Sample: N vs $N-1$

- ▶ If these were true population probs, then divide this sum by $N=4$, which gives you 87.5.
 - ▶ This translates into an sd of 9.35%.
- ▶ If these were just historical samples, then divide sum by $N-1=3$, which gives you 116.67.
 - ▶ This translates into an sd of 10.8%.

Excel

- ▶ Excel *stdev()* uses $N-1$.
 - ▶ Excel assumes you knew only the sample.
- ▶ This makes little difference for large datasets.
 - ▶ But, it matters with $textN = 4$.
- ▶ (N vs $N - 1$ matters only for small samples.)
 - ▶ Excel: *stdevp()* is not *stdev()*;
 - ▶ Excel: *varp()* is not *var()*.

Repeat Conceptual Warning

- ▶ Do not trust the historical means blindly for predicting future expected RoRs.
- ▶ For individual stocks (rather than big diversified portfolios), this would be exceedingly stupid.
- ▶ Even for big diversified portfolios (like the S&P 500), this is a big leap of faith.

BUT NOT ALL IS LOST

- ▶ (Recent) historical variances (and covariances and standard deviations) are usually good predictors of (short-term) future variances (and covariances and standard deviations).
- ▶ Advice: use 1–3 years of historical *daily* return data to estimate them.

Estimating Tail Risk?

- ▶ *Unfortunately*, a short time-series of historical numbers is usually not reliable enough to calculate tail-risk—the probability of a complete blow-up.
 - ▶ How do you estimate the risk of the next Space-X rocket exploding?
 - ▶ How do you estimate the prob of the next meteor hitting NYC from 80 years of data?
 - ▶ We would need more data than we have!
 - ▶ (Nerd Technical Alternative: Use [Options](#).)
- ▶ Shrink them.

Monthly Stock Returns By Decade

Decade	Avg Ret	SD(Ret)
1990s	13%	13.4%
2000s	-4%	16.1%
2010s	13%	12.5%

Big Context Switch

Investor Preferences

Preferences over Bets

- ▶ If I offer you a bet of +\$1 if heads and -\$1 if tails, you pick a coin and someone else in class to throw it (at least 5 yards!), would you be willing to take this bet?
- ▶ If not, how much would I have to pay you?

Low Risk-Aversion

When is risk neutrality (or low risk-aversion) a good assumption?

Risk

Why do people climb mountains, play the lottery, drive motorcycles, or fly (Vans RV) airplanes?

Risk Preferences

- ▶ **Most of finance assumes that investors are risk-averse.**
- ▶ **When choosing between overall portfolios with equal expected returns, investors prefer the lower-risk one.**
- ▶ **Portfolio Risk can be measured by portfolio SD.**

Enough Theory

- ▶ now come applications of these concepts...!