

Investing Choices and Risk Measures

(Welch, Chapter 08)

Ivo Welch

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Maintained Assumptions

Perfect Markets

1. No differences in opinion.
2. No taxes.
3. No transaction costs.
4. No big sellers/buyers—infininitely many clones that can buy or sell.

With risk and risk aversion

First Question: Investors

- ▶ **How should I choose among many different projects?**
 - ▶ taking market prices as fixed and given.

Second Question: Corporate Managers

- ▶ How do (your) projects determine company risk?
- ▶ How do (your) investors think?
- ▶ What will make them want to give you money?
- ▶ What is *your* opportunity cost of capital $E(r)$?

Investor Risk Preferences

- ▶ They care only about their overall portfolio, not about your own or any one project per se.
 - ▶ Investors don't care about your or any other specific project.
 - ▶ Your own project's SD may or may not matter to them. We will have to figure this one out.
- ▶ We assume investors care about the standard deviation (SD) of their *overall portfolio* return.

Investments

- ▶ Four equally likely scenarios (“states”):
 - ▶ states: yellow, red, green, blue.
 - ▶ Nerddnote: “state-based” preferences are more general than our Mean/SD preferences.
- ▶ Four investment assets: A, B, C, D.
- ▶ Returns (in Percent or Dollars).

Investment Contingencies

	Ylw	Red	Grn	Blu
A:	-4.0	-4.0	+6.0	+6.0
B:	-1.0	+9.0	+9.0	-1.0
C:	-1.25	+1.25	+3.75	+1.25
D:	+3.0	+13.0	+3.0	-7.0

Investment Rewards

- ▶ What are the rewards of the four investments?
 - ▶ We measure “rewardiness” as expected rate of return.
 - ▶ “Risk and reward” usually means “expected standard deviation and expected mean.”
 - ▶ PS: The standard deviation is a known number today. Thus standard deviation and *expected* standard deviation and *exp. exp.* standard deviation are all the same thing.

Investment Risks

- ▶ What are the risks of the four investments?

Population vs Sample Statistics

- ▶ If Ylw-Blu returns are just representative historical realizations, you would divide by 3, not 4 in your computation of the variance.
- ▶ In real life, we rarely have population statistics.
 - ▶ Historical data are sadly often our best choice,
 - ▶ but *here*, for learning, we assume we know the distribution.

Overall vs Parts Risk

The standard deviation is a meaningful measure of risk only for your overall portfolio.

We need not care about the standard deviations of individual investments.

Means and SDs of 4 Assets

	Mean	X-Mean	Var	SD
A:				
B:				
C:				
D:				

Portfolio Risk

- ▶ What is the risk of an (equal-weighted) portfolio of Asset A and Asset B?
 - ▶ (HINT: First compute the RoRs of the combination portfolio in each state.)

Portfolio Risk

- ▶ Is the average portfolio or are the individual components riskier?
- ▶ Why?

Good Portfolios?

- ▶ What kind of portfolio would you — a smart but risk-averse investor — hold?

Real Life Prime Portfolio

- ▶ In real life, what portfolios should and do smart investors with risk-aversion hold?

Portfolio Risk

- ▶ What is A's portfolio risk if you add C to your portfolio vs if you add D to your portfolio?

	Mean	X-Mean	Var	SD
A+C:				
B+D:				

Riskier Investment I

- ▶ Is C or D the riskier investment in itself?

Riskier Investment II

- ▶ If you already own A, is C or D the riskier addition?

Why?

Base Portfolio

- ▶ If investors are smart, what is their base portfolio A?

CAPM Preview

- ▶ **Previewed Guess:** If you are selling to smart investors either C or D, for which of these two projects do you think will investors clamor to invest in your project?
 - ▶ More clamoring : they will accept lower exp. RoR.
 - ▶ Lower exp. RoR : they will pay a higher price.

Fundamental Investment Insight

- ▶ Investors (should) care about overall portfolio risk, not about the constituent component risk.
- ▶ From a corporate managerial perspective, it is not low-risk projects that investors like, **BUT** projects which wiggle opposite to the rest of their portfolios.

Synchronicity

- ▶ How should we measure synchronicity?
 - ▶ For exposition, consider A to be the market portfolio that investors are already holding.
 - ▶ We need a measure of how synchronous or non-synchronous any new stock/asset/project is with respect to this portfolio $A=M$.

Calculate Ri-Mean(Ri)

| Ylw | Red | Grn | Blu |

|---+---+---+---+ | A: | -5.0 | -5.0 | +5.0 | +5.0 | |
B: | -5.0 | +5.0 | +5.0 | -5.0 | | C: | -2.5 | 0.0 | +2.5 | 0.0 | | D:
| 0.0 | +10.0 | 0.0 | -10.0 |

Calculate COVAR

- Covariance is mean of cross-products:

$$(A - \bar{A}) \times (B - \bar{B}) = +25, -25, +25, -25.$$

$$(A - \bar{A}) \times (C - \bar{C}) = +12.5, 0, +12.5, 0.$$

$$(A - \bar{A}) \times (D - \bar{D}) = +0, -50, 0, -50.$$

$$\text{cov}(A,B) = 0, \quad \text{cov}(A,C) = +6.25, \quad \text{cov}(A,D) = -25.$$

- why are we demeaning and multiplying?

Calculate Beta (Slope)

- ▶ Beta is the covariance divided by the variance:

$$\beta_{C,A} = 6.25/25 = 0.25 ,$$

$$C = 1 + 0.25 \cdot A, \quad (A = -1.5 + 2 \cdot C) .$$

- Ergo, beta of C (y) on A (x) is 0.25. “A-beta of C”

$$\beta_{D,A} = -25/25 = -1,$$

$$D = 4 - 1 \cdot A, \quad (A = 2.5 - 0.5 \cdot D) .$$

Calculate Correlation

- ▶ The correlation is the covariance divided by the standard deviations of its two ingredients:

$$\text{cor}(A,C) \approx 0.7071 \quad \text{cor}(A,D) \approx -0.7071 .$$

- ▶ The order does not matter for covariance or correlation. It matters only for beta.

Risk Contribution Measures?

- ▶ Covariance generalizes variance. (Why?) Thus, it also has uninterpretable units. Yuck.
- ▶ Correlation has a scale problem:
 - ▶ Think you own one investment paying -1% or $+1\%$.
 - ▶ An investment with -0.01% or $+0.01\%$ has perfect corr (1.0).
 - ▶ An investment with -100.0% or $+100.0\%$ has perfect corr (1.0).
 - ▶ Which one would contribute more risk?

Best Measure: Market-Beta

- ▶ The best risk contribution of adding B to M is B's market-beta with respect to M.
 - ▶ This means $var(R_m)$ is the denominator.
 - ▶ Without verbal qualification, beta always means with respect to R_m , i.e., market-beta.
 - ▶ Market-beta is the “M-beta” of asset B.

Happy Family

- ▶ Covariance, correlation, and beta have the same sign.
- ▶ They differ only in magnitude.

Beta is a Line Slope

- ▶ Beta is a slope. Put A (M) on the X axis, and your project B (or C) on the Y axis.
 - ▶ A slope of 1 is a diagonal line.
 - ▶ A slope of 0 is a horizontal line.
 - ▶ A slope of ∞ is a vertical line.
- ▶ Without alpha, beta tells you how an x% higher RoR (than normal) in the market will likely associate with simultaneously higher RoR (than normal) in your stock.
 - ▶ think $\beta_i \cdot x\%$ higher rate of return (than normal)

Beta Interpretation

- ▶ The stock-return beta helps with a conditional forecast of R_i , given R_m .
 - ▶ If you can go short the market, you can even (on average) reduce your portfolio's net market exposure down to 0. Beta tells you how short you should be.

Beta Measurement

- ▶ Mediocre measures of market-beta are available on every finance website.
 - ▶ A better estimator would be to use daily stock returns on 1-2 years of historical data.
 - ▶ The **best market-beta measure** shrinks smartly.
 - ▶ winsorizing means trimming to limits.
 - ▶ Do not use other stocks' betas if you do not have to.

Market-Beta of Market

- ▶ What is the market-beta of the overall stock market (say, the S&P500)?

Market-Beta of Risk-Free Rate

- ▶ What is the market-beta of the risk-free rate?

High vs Low *Beta* Projects

- ▶ Given equal expected returns, what's more desirable to an investor heavily loaded up on the stock market?
 - ▶ A project with a high beta? Or
 - ▶ A project with a low beta?

High vs Low *Risk* Projects

- ▶ Should high or low variance projects have to offer higher expected RoRs?

High vs Low *Beta* Projects

- ▶ Should high or low beta projects have to offer higher expected RoRs?

Conglomeration

- ▶ New Firm: 40% C and 60% D. (\$4m and \$6m.)
- ▶ What is the average RoR (mean)?
- ▶ What is the average variance?
- ▶ What is the average sd?
- ▶ What is the average beta?

Value-Averaging

- ▶ Which statistics can you “value-average”?
- ▶ Which statistics can you not “value-average.”

Corporate Market Beta

- ▶ Is there a quicker way to compute the overall market-beta of your firm, based on the market-betas of its constituent projects?

Nerd Warning: Time-Changing Pftio Weights

- ▶ Firms in portfolios constantly change their relative investment weights:
 - ▶ In historical data use, the weighted formula requires fixed weights.
 - ▶ Because more appreciating stocks get more weight, this often means that you cannot use today's investment weights in a historical calculation.
 - ▶ Put differently, your portfolio weights and overall beta can change all the time.

Mean-Variance Frontier

(Omitted, but some discussion in book.)

- ▶ The mean-variance efficient frontier (= the mean-standard deviation efficient frontier).
 - ▶ optimal combination of assets.
 - ▶ covering it would require 2+ full lectures.
 - ▶ Underlies CAPM+. Take an investments course!
 - ▶ It is in common (practical) use.
 - ▶ Important.

Variance of Weighted Sum

If portfolio P consists of two assets, A and B :

$$r_P = w_A \cdot r_A + w_B \cdot r_B ,$$

then the portfolio variance is

$$\begin{aligned} \text{Var}(r_P) &= \text{Var}(w_A \cdot r_A + w_B \cdot r_B) = \\ &w_A^2 \cdot \text{Var}(r_A) + w_B^2 \cdot \text{Var}(r_B) + \\ &2 \cdot w_A \cdot w_B \cdot \text{Cov}(r_A, r_B) . \end{aligned}$$

- ▶ This “ $2 \cdot w_A \cdot w_B \cdot Cov(r_A, r_B)$ ” term means it is *not* the weighted average, “ $w_A \cdot Var(R_A) + w_B \cdot Var(R_B)$ ”!
- ▶ One cannot value-weight variances!
 - ▶ A portfolio of half A and half B usually does not have a variance that is the average of the variances of A and B.

Effect of Changing Weights

- ▶ The generalized variance formula is based on variance-covariance matrix between all assets and all your investment weights.
- ▶ It makes it easier to recompute the portfolio risk when you change portfolio weights.

Time Correlation

- ▶ What is the expected correlation of stocks' RoRs from one day to the next day?

Time Correlation

- ▶ Can the expected correlation be a little off from 0?

Time Correlation

- ▶ Can the *actual* realized correlation be a little off from 0?

Time-Adjusting Risk

- ▶ This is an important application of the variance-covariance formula.
- ▶ If the risk of investing in x for 1 year is $\sigma=20\%$, what is the risk of investing for 10 years?

A1: Constant Risk

- ▶ Let's assume that the per-unit-of-time standard deviation remains constant.
 - ▶ This need not be the case.
 - ▶ It is an additional assumption.
 - ▶ Risk before harvest may be different than risk before planting.
 - ▶ We can then omit time subscript.
 - ▶ We call this constant annual number σ .

A2: Uncorrelated over Time I

- ▶ Rates of return over time should be uncorrelated
 - ▶ This is a consequence of a perfect market.
 - ▶ Otherwise, you could use past stock returns to outpredict future stock returns.
 - ▶ If not exactly zero, likely just a little statistical noise.
- ▶ Algebraically, for $s \neq 0$,

$$\text{Cov}(R_t, R_{t+s}) \approx 0 ,$$

where the subscripts t and $t + s$ refer to two time periods, not to different stocks at the same time.

A2: Uncorrelated over Time II

- ▶ In this case, the following approximation is not bad:

$$Sd(R_{0,T}) \approx \sqrt{T} \cdot \sigma$$

- ▶ **Example:** if your portfolio risk is 10% per month, then your annual risk is about $\sqrt{12} \cdot 10\% \approx 35\%$ per year.

Time-Adjusted Derivation

$$\begin{aligned} \text{Var}(R_{0,T}) &\approx \text{Var}(R_{0,1} + R_{1,2} + \dots + R_{T-1,T}) \\ &= \text{Var}(R_{0,1}) + \text{Var}(R_{1,2}) + \dots + \text{Var}(R_{T-1,T}) \\ &= \text{many } 0 \text{ covariance terms} \\ &\approx T \cdot \sigma . \end{aligned}$$

Sharpe Ratio

- ▶ **Sharpe-Ratio (SR):** a (badly flawed but common) measure of investment performance:



$$SR_i = \frac{\text{Avg}(R_i - R_f)}{SD(R_i - R_f)} \approx \frac{\text{Avg}(R_i) - R_f}{SD(R_i)} .$$

- ▶ The SR grows with the square-root of time.
- ▶ Calculated typically from *monthly* RoRs **annualized** by $\sqrt{12}$.
- ▶ Historical SR of Stock Market: 4%/10% \approx 0.4.

Nerd: VW/EW Portfolio Maintenance

- ▶ Is it easier to maintain a value-weighted or an equal-weighted portfolio?